

## Natural time analysis: Important changes of the order parameter of seismicity preceding the 2011 M9 Tohoku earthquake in Japan

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Abstract – Applying the natural time analysis to the Japanese seismic data, we find that almost three months before the 11 March 2011 M9 Tohoku earthquake the order parameter fluctuations of seismicity exhibited an abrupt increase with a natural time scale dependence that has a functional form strikingly reminiscent of the one discussed by Penrose and coworkers in computer simulations of phase transition kinetics using the ideas of Lifshitz and Slyozov. This increase on 22 December 2010, which is shown to be of profound statistical significance by employing the recent method of event coincidence analysis, accompanies increased fluctuations of the entropy change of seismicity under time reversal that obey the Lifshitz-Slyozov-Wagner theory for phase transitions. On the same date the entropy change of seismicity under time reversal is minimized, while the order parameter fluctuations of seismicity exhibit a minimum almost two weeks later.

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Introduction. - Earthquakes (EQs) exhibit in general complex correlations in time, space and magnitude, e.g., [1-5]. It is widely accepted [6,7] that the observed EQ scaling laws indicate the existence of phenomena closely associated with the proximity of the system to a critical point. In particular, it has been indicated by Carlson et al. [6] that it seems possible that systems that operate persistently near a threshold of instability are in some way like thermodynamic systems near critical points (EQ can be regarded as a stick-slip frictional instability of a pre-existing fault). The order parameter of seismicity is the quantity by which one can identify the approach of the dynamical system to a critical point. The introduction of such a parameter for the case of seismicity, labeled hereafter  $\kappa_1$  became possible after the suggestion of natural time analysis (see the next section) which uncovers important hidden properties in time series of complex systems [8] and has been recently employed by Turcotte and coworkers as a basis of a new methodology to estimate the current seismic risk level [9–15].

Upon analyzing the Japan seismic catalogue in natural time and computing the fluctuations of  $\kappa_1$  interesting results have been obtained. To compute  $\kappa_1$  fluctuations, the

procedure explained in refs. [16,17] was applied by using a sliding natural time window comprising the number *i* of EQs that would occur on average within the lead time of Seismic Electric Signals (SES) activities, which are series of low-frequency transient changes of the electric field of the Earth preceding EQs [18,19], *i.e.*, within the crucial scale [20] of a few months, or so. We then calculate the average value  $\mu_i(\kappa_1)$  and the standard deviation  $\sigma_i(\kappa_1)$ of the ensemble of  $\kappa_1$  obtained (see the "Data analysis" section). The quantity

$$\beta_i \equiv \sigma_i(\kappa_1) / \mu_i(\kappa_1) \tag{1}$$

is defined [8] as the variability of  $\kappa_1$ . The time evolution of the  $\beta$  value can then be pursued by sliding the excerpt of length *i* (which will be hereafter alternatively called "scale") through the EQ catalogue and the corresponding minimum value is labeled  $\beta_{min}$ . The following key-results have been obtained.

Sarlis *et al.* [16] analyzed the Japan seismic catalog in natural time from 1 January 1984 to 11 March 2011. The results showed that the fluctuations  $\beta$  of the order parameter of seismicity exhibited distinct minima  $\beta_{min}$ a few months before all the shallow EQs of magnitude 7.6 or larger that occurred during this 27-year period in the Japanese area  $N_{25}^{46}E_{125}^{148}$ . Among these minima, the minimum before the M9 Tohoku EQ observed around 5 January 2011 was the deepest.

It is the main scope of this paper to investigate what happens with the fluctuations  $\beta$  of the order parameter of seismicity before the appearance of the aforementioned deepest minimum  $\beta_{min}$  on 5 January 2011. (In general, in phase transitions the order parameter close to the critical state is expected to undergo non-Gaussian fluctuations, but almost nothing is known [21] about the mathematical form of the possible probability distributions of the order parameter except for a few cases [21,22], thus any result to understand which kind of fluctuations the order parameter can experience at criticality is of chief importance.) This investigation is challenging because, very recently we have shown [23,24] that almost two weeks earlier, *i.e.*, on 22 December 2010, the following two additional facts have been observed: First, the entropy change of the seismicity under time reversal is minimized [24]. Second, the fluctuations of the entropy change of seismicity under time reversal exhibited [23] an abrupt increase which conforms to the seminal work by Lifshitz and Slyozov [25] and independently by Wagner [26] for phase transitions. These authors derived some exact results in the limit that the minority phase occupies a negligible volume fraction (e.g.,see pp. 370–371 of ref. [27]) and in particular they showed that the characteristic size of the minority phase droplets exhibits a scaling behavior in which time growth has the form  $A(t-t_0)^{1/3}$ . In ref. [23] it was also found that the increase  $\Delta \Lambda_i$  of the complexity measure  $\Lambda_i$  quantifying these fluctuations follows the latter form and that the prefactors A are proportional to the scale *i*, while the exponent (1/3)is independent of i. (This form of time growth is more or less reminiscent of eqs. (100.14) and (100.23) in the chapter entitled "Kinetics of Phase Transitions" of Vol. 10 of Landau and Lifshitz Course of Theoretical Physics [28]. Note, however, that the theory which leads to these two equations does not take into account fluctuations of the order parameter. Its applicability is therefore restricted by the same conditions as for the Landau thermodynamic theory of phase transitions. These conditions are not satisfied in a neighborhood of the transition point, the "fluctuation" region, see p. 441 of ref. [28].)

In view of these recent findings the present paper is structured as follows: In the next two sections we recapitulate the basic procedure for the natural time analysis of seismicity in general and for the data analysis. In the subsequent section we present the new findings on what happened for the order parameter fluctuations of seismicity on 22 September 2010. A discussion follows and finally in the last section we summarize our conclusions.

Natural time analysis of seismicity. Background. – In a time series comprising  $N \in Qs$ , the natural time for the occurrence of the k-th EQ of energy  $Q_k$  is defined as  $\chi_k = k/N$ . In natural time analysis, we study the

evolution of the pair  $(\chi_k, p_k)$ , where

$$p_k = Q_k / \sum_{n=1}^N Q_n \tag{2}$$

denotes the normalized energy released during the k-th EQ.  $Q_k$  and hence  $p_k$  for EQs is estimated through the relation [29]

$$Q_k \propto 10^{1.5M_k} \tag{3}$$

Varotsos et al. [30] argued that the variance

$$\kappa_1 = \langle \chi^2 \rangle - \langle \chi \rangle^2 \tag{4}$$

of natural time  $\chi$  weighted for  $p_k$ , given by

$$\kappa_1 = \sum_{k=1}^{N} p_k(\chi_k)^2 - \left(\sum_{k=1}^{N} p_k \chi_k\right)^2,$$
 (5)

may serve as an order parameter of seismicity.

The entropy S in natural time is defined [31] by

$$S \equiv \langle \chi \ln \chi \rangle - \langle \chi \rangle \ln \langle \chi \rangle \tag{6}$$

where the brackets  $\langle \ldots \rangle \equiv \sum (\ldots) p_k$  denote averages with respect to the distribution  $p_k$ , *i.e.*,  $\langle f(\chi) \rangle \equiv \sum f(\chi_k)p_k$ . Notably, the functional given by eq. (6) has been shown [32] to exhibit positivity, concavity, and experimental stability, which are the three requirements in order to be characterized as entropic functional. Furthermore, note that the entropy S is a dynamic entropy depending on the sequential order of the events and not simply a statistical entropy (*e.g.*, Shannon entropy), see ref. [33]. Upon considering time reversal  $\hat{T}$ , *i.e.*,  $\hat{T}p_k = p_{N-k+1}$ , the value S changes to a value  $S_{-}$ :

$$S_{-} = \sum_{k=1}^{N} p_{N-k+1} \left(\frac{k}{N}\right) \ln\left(\frac{k}{N}\right)$$
$$- \left(\sum_{k=1}^{N} \frac{k}{N} p_{N-k+1}\right) \ln\left[\sum_{l=1}^{N} \frac{l}{N} p_{N-l+1}\right]. \quad (7)$$

The physical meaning of the entropy change  $\Delta S \equiv S - S_{-}$  in natural time under time reversal is discussed in refs. [8,34] using the distribution  $p(\chi; \epsilon) = 1 + \epsilon(\chi - 1/2)$  which replaces  $p_k$  when considering a continuous variable  $\chi \in (0, 1]$  instead of  $\chi_k$ . Small  $|\epsilon| < 1$  represents an increase ( $\epsilon > 0$ ) or decrease ( $\epsilon < 0$ ) of  $Q_k$  when k increases, thus reflecting the effect of small linear trends in  $Q_k$ . It can be shown that  $\Delta S(\epsilon) = \frac{(6 \ln 2 - 5)}{36} \epsilon + O(\epsilon^3)$  leading to the conclusion that a small increasing trend leads to negative  $\Delta S$  and vice versa.

Using a moving window of length i (number of events) sliding through the time series of L consecutive events the entropy in natural time is determined for each position j = $1, 2, \ldots, L-i$  of the sliding window. Thus, a time series of  $(S_i)_j$  is obtained. By considering the standard deviation



Fig. 1: Plot of the fluctuations  $\beta_{300}$  of the order parameter of seismicity vs. the conventional time for i = 300 events. Period from 1 January 1984 until the M9 Tohoku EQ occurrence on 11 March 2011. The shallow EQs with  $M \ge 7.0$  are also shown (right scale) with vertical bars ending at asterisks. The precursory change on 22 December 2010 discussed for the first time in this paper is shown with an arrow. The descriptive statistics of  $\beta_{300}$  have been presented in fig. 3(f) of ref. [35] where one can read that the mean value is 0.55 while the standard deviation is 0.23. In view of the fact that the maximum value attained on 22 December 2010 is around 1.35, we find that this maximum corresponds to a fluctuation of around 3.5 standard deviations.

 $\sigma(\Delta S_i)$  of the time series of  $(\Delta S_i)_j \equiv (S_i)_j - [(S_-)_i]_j$ , we define [8,36,37] the complexity measure  $\Lambda_i$ , which constitutes a measure for the entropy change fluctuations under time reversal,

$$\Lambda_i = \frac{\sigma(\Delta S_i)}{\sigma(\Delta S_{100})},\tag{8}$$

where the denominator has been selected [36] to correspond to the standard deviation  $\sigma(\Delta S_{100})$  of the time series of  $(\Delta S_i)_j$  of scale i = 100 events.

 $\Delta S$  constitutes a key measure that may identify when the system approaches the critical point (dynamic phase transition) [8]. For example,  $\Delta S$  has been applied [34] for the identification of the time of an impending sudden cardiac death risk. Furthermore, it has been used [38] for the study of the predictability of the Olami-Feder-Christensen (OFC) model for EQs [39], which is probably [40] the most studied non-conservative self-organized criticality (SOC) model. The OFC model originated by a simplification of the Burridge and Knopoff spring-block model [41] by mapping it into a non-conservative cellular automaton simulating the EQ's behavior and introducing dissipation in the family of SOC systems. In particular, it was found that  $\Delta S$  exhibits a clear minimum [8] (or maximum if we define [38]  $\Delta S \equiv S_{-} - S$  instead of  $\Delta S \equiv S - S_{-}$ ) before a large avalanche in the OFC model, which corresponds to a large EQ. For example, by analyzing the seismicity during the period 2012-2017 in natural time in the Chiapas region of Mexico, where the M8.2 EQ occurred on 7 September 2017, we observed [37] that on 14 June 2017, the entropy change  $\Delta S$  of seismicity under time reversal was minimized.

**Data analysis.** – The method for our present calculations on natural time analysis of seismicity in Japan could be recapitulated as follows: In natural time analysis of EQs, the quantity  $Q_k$  is estimated as mentioned through the usual relation [29]  $Q_k \propto 10^{1.5M_{w,k}}$ , where  $M_{w,k}$  is the moment magnitude of the k-th EQ. For the data presented here that come from the Japan Meteorological Agency (JMA), the formulae suggested by Tanaka et al. [42] have been used for the conversion of the magnitude  $M_{JMA}$  reported by JMA to moment magnitude. For data completeness a magnitude threshold has been adopted [16] by considering all EQs with  $M_{\rm JMA}$  or simply  $M \geq 3.5.$  Let us now consider an excerpt of a seismic catalog comprising iconsecutive events and construct all the sub-excerpts of 6 to *i* consecutive events. The computed  $\kappa_1$  values for all these sub-excerpts (cf. at least 6 events are required [30] for a reliable estimation of  $\kappa_1$ ) enable the calculation of their average value  $\mu_i(\kappa_1)$  and their standard deviation  $\sigma_i(\kappa_1)$ . We can then determine the variability  $\beta$  of  $\kappa_1$ ,  $\beta_i \equiv \sigma_i(\kappa_1)/\mu_i(\kappa_1)$ , which corresponds to this excerpt of *i* EQs. To compute the time evolution of  $\beta_i$  we apply the above procedure by sliding (each time by one event) an overlapping moving window of length of i EQs, estimate  $\beta_i$  within this window and assign the corresponding value to the occurrence time of the first EQ (target EQ) after the completion of the excerpt. Hence, for the  $\beta_i$  value of a target EQ only its past EQs are used in the calculation.

The new precursory change of the order parameter fluctuations of seismicity before the 2011 M9 **Tohoku EQ.** – In fig. 1 we plot the fluctuations  $\beta$  of the order parameter of seismicity in the entire Japanese region  $N_{25}^{46}E_{125}^{148}$  vs. the conventional time from 1 January 1984 until the Tohoku EQ occurrence on 11 March 2011 upon considering all M > 3.5 EQs irrespectively of their depth. The sliding natural time window comprised 300 events, which is the average number of EQs ( $M \ge 3.5$ ) that occur within  $\approx 2$  months. All shallow EQs with magnitude 7 or larger are also shown with vertical black bars ending at asterisks (right scale). A careful inspection of this figure shows that there exist six prominent fluctuations of  $\beta$  (higher than 1.2) before the M9 Tohoku EQ. These are accompanied by major EQs and can be better seen in fig. 2 where excerpts of fig. 1 in expanded horizontal scale



Fig. 2: Excerpts of fig. 1 in expanded horizontal scale. (a)–(e) They show what happens upon the occurrence of each of the major shallow EQs ( $M \ge 7.6$ ) in Japan during the period 1 January 1984 until the M9 Tohoku EQ. (f) The change observed after the M7.5 EQ (not shown since it is not shallow) on 15 January 1993. Panels (g) and (h) depict the lin-lin and lin-log plots, respectively, of the interrelation  $\Delta \beta_i = A \ln(i/B)$  between the variation  $\Delta \beta_i$  of the increase of  $\beta_i$  upon the occurrence of the M7.8 EQ on 22 December 2010 vs. the window length *i*, for i = 150-500 events. The least squares fit results in  $A = 0.500\pm0.012$  and  $B = 114.3\pm2.9$  events.

are depicted. These include the fluctuation observed at the beginning of 1993 after the M7.5 EQ on 15 January 1993 at 42.92° N 144.35° E and the fluctuations of  $\beta$  associated with all shallow EQs of magnitude 7.6 or larger during the period investigated which are the following: The Southwest-Off Hokkaido M7.8 EQ on 12 July 1993, the East-Off Hokkaido M8.2 EQ on 4 October 1994, the Far-Off Sanriku M7.6 EQ on 28 December 1994, the Off Tokachi M8.0 EQ in 2003, and the Near Chichi-jima M7.8 EQ in 2010. As it becomes evident from fig. 1, the two larger fluctuations are in 2003 and in 2010: First, the large fluctuation of  $\beta$  in 2003 appears upon the occurrence of the M8 Off Tokachi EQ on 26 September 2003 (and attains its maximum value on 28 September 2003, see fig. 2(b)). It is the largest one observed upon the occurrence of any other major EQ during the 27-year period of our study. Second, the fluctuation of  $\beta$  on 22 December 2010 is observed upon the occurrence on the same day of the M7.8 near Chichi-jima EQ. The following comments are now in order concerning these two larger  $\beta$  fluctuations. The second, *i.e.*, the one in 2010, appears almost simultaneously with the minimum  $\Delta S_{min}$  of the change  $\Delta S$  of the entropy of seismicity under time reversal which occurs also on 22 December 2010 as it was found upon applying the procedure of ref. [24] to the entire Japanese region seismic data, where we also demonstrate that the probability to obtain such a minimum by chance is approximately 3% thus showing that it is statistically significant. In addition, the robustness of the appearance of this minimum on 22 December 2010 upon changing the EQ depth, the EQ magnitude threshold, and the size of the area investigated has been documented [24]. Recall that such a minimum is of precursory nature signaling that a large EQ is impending according to the conclusions deduced from the natural time analysis of the OFC model summarized in the last paragraph of the "Natural time analysis of seismicity. Background" section. In addition, on the same day the complexity measure  $\Lambda_i$  associated with the fluctuations of the entropy change of seismicity under time reversal showed [23] an increase, as mentioned. This increase is also of precursory nature since it exhibits a scaling behavior [23], which conforms to the seminal work by Lifshitz and Slyozov [25] and independently by Wagner [26] on phase transitions, as already mentioned (such a behaviour is not obeyed [23] by the  $\Delta \Lambda_i$  increases observed upon other EQ occurrences, e.g., on 2 November 1989 and 15 January 1993, at which increases of  $\beta$  fluctuations are also observed in fig. 1). These facts corroborate in the



Fig. 3: Comparison of the variation  $\Delta\beta_i$  of the increase of  $\beta_i$ upon the occurrence of the M7.8 EQ on 22 December 2010 vs. the window length *i*, for i = 150-500 events when considering all  $M \geq 3.5$  EQs, *i.e.*, irrespectively of their depth *h* (red solid dots) —as depicted in panel (h) of fig. 2— with that obtained when restricting ourselves either to shallow EQs with depth  $h \leq 70 \,\mathrm{km}$  (green circles) or to shallow and intermediate depth EQs with  $h \leq 300 \,\mathrm{km}$  (green squares). The green horizontal line has been drawn as a guide to the eye.

suggestion that between the two larger  $\beta$  fluctuations discussed here only the second, *i.e.*, the one on 22 December 2010, that precedes the Tohoku M9 EQ by almost three months, is likely to be of precursory nature. Note that the validity of this conclusion has been also checked for other lengths from i = 150 to 500 events, instead of the case i =300 events presented in fig. 1. Such a change of the length ialso reveals the following behavior: Upon increasing i it is observed (see figs. 2B and 4E of ref. [16]) that the increase  $\Delta \beta_i$  of the  $\beta_i$  fluctuation on 22 December 2010 becomes distinctly larger which does not happen (see figs. 4A-D of ref. [16]) for the increases of the  $\beta$  fluctuations upon the occurrences of all other shallow EQs in Japan of magnitude 7.6 or larger during the period from 1 January 1984 to the time of the M9 Tohoku EQ. Such a behavior that obeys the interrelation  $\Delta \beta_i = 0.5 \ln(i/114.3)$ , see fig. 2(g) and (h), has a functional form strikingly reminiscent of the one discussed by Penrose et al. [43] in computer simulations of phase separation kinetics using the ideas of Lifshitz and Slyozov [25], see their equation (33) which is also due to Lifshitz and Slyozov (it is important to note that this functional form is lost upon considering in our calculation not all M  $\geq 3.5$  EQs but restrict ourselves to depths h either  $h \leq 70$  km or  $h \leq 300$  km, see the symbols in green in fig. 3). Hence, the  $\beta$  fluctuation on 22 December 2010 accompanying the minimum  $\Delta S_{min}$  is unique. By employing the most recent method of Event Coincidence Analysis [44] (ECA), which considers a time lag  $\tau$ and a window  $\Delta T(>0)$  between the precursor and the event to be predicted, the profound statistical significance of this unique result can be further assured as follows: Assuming the M9 Tohoku EQ as the event to be predicted, we estimated the probability (*p*-value) that the increased fluctuation on 22 December 2010 is a chancy precursor

with a lag  $\tau$  and a window  $\Delta T$ , *i.e.*, the EQ occurs within the time period from  $\tau$  to  $\tau + \Delta T$  days after. By employing the function CC.eca.es of the CoinCalc package [45] for R that implements ECA, we found that the probability for this to happen by chance is very small, *i.e.*, it varies from p = 0.79% to 0.01% when the window  $\Delta T$  varies from 78 days to 1 day, respectively.

Discussion. – We found above that on 22 December 2010 an increase of the fluctuations of the order parameter of seismicity appeared, the scale dependence of which exhibits a functional form reminiscent of the one discussed by Penrose *et al.* in computer simulations of phase transition kinetics using the ideas of Lifshitz and Slyozov. We recall that on the same date the entropy change of seismicity under time reversal is minimized along with increased fluctuations that obey the Lifshitz-Slyozov-Wagner theory for phase transitions. Two weeks later, *i.e.*, around 5 January 2011, an unprecedented minimum  $\beta_{min}$  of the fluctuations of the order parameter of seismicity appeared as mentioned in the Introduction. This was observed almost simultaneously with the initiation of anomalous magneticfield variations mainly in the z-component [46-48], which reflects the initiation of a strong SES activity being consistent with earlier results in Greece where major EQs have been found to be preceded by intense SES activities —with lead time ranging from a few weeks to  $5\frac{1}{2}$  months [8]— accompanied by clear Earth's magnetic-field variations [49] mainly recorded on the z-component [50]. Notably, an early model [51] for the SES generation foresees a phase change (and in particular a second-order dynamic phase transition) as follows: In the Earth, in the future focal region of an EQ, electric dipoles always exist due to the lattice imperfections [52] (point and linear defects [53]) in the ionic constituents of rocks, which have initially random orientations. The stress  $\sigma$  starts to gradually increase due to an excess stress disturbance and when this stress reaches a *critical* value ( $\sigma_{cr}$ ) the electric dipoles exhibit a cooperative orientation (note that cooperativity is a hallmark of criticality) resulting in the emission of a transient electric signal SES.

**Conclusions.** – Our main conclusion could be summarized as stating that at the date 22 December 2010 at which we previously reported [24] that the entropy change of seismicity under time reversal is minimized, the order parameter fluctuations (that are minimized two weeks later) showed a unique change, *i.e.*, an abrupt increase which exhibits a functional form consistent with the ideas of Lifshitz and Slyozov for the time growth of the characteristic size of the minority phase droplets in phase transitions.

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