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Precursory variations of Tsallis non-extensive statistical mechanics entropic index associated with the M9 Tohoku earthquake in 2011

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Abstract. Applying natural time analysis, which has been introduced by the authors in 2001, to the Japanese seismic data, we find that the system enters the critical stage upon the occurrence of $M = 4.2$–5.0 earthquakes from 08:36 to 13:14 local time (LT) on 10 March 2011, i.e., almost one day before the $M9$ Tohoku earthquake on 11 March 2011. Here, we find that just before this period the Tsallis entropic index q of non-extensive statistical mechanics started to show distinct changes. In addition an evident change of q is found upon the occurrence of the $M7.3$ foreshock at 11:45 LT on 9 March 2011, which exhibits a scaling behavior with a characteristic exponent 1/3 that conforms to Lifshitz-Slyozov-Wagner theory for phase transitions.

1 Introduction

Non-extensive statistical mechanics [1], pioneered by Tsallis [2,3], provides a framework for the study of complex systems in their non-equilibrium stationary states, as well as in systems with (multi)fractal and self-similar structures, long-range interacting systems, etc. It is a generalization of the classical statistical theory of Boltzmann and Gibbs and the involved entropy is a mono-parametrical function of the probability distribution. This entropic parameter q can attain any value, while for $q \to 1$, recovers the Boltzmanian entropy and the Boltzmann-Gibbs statistical mechanics. The non-extensive statistical mechanics is the background of kappa distributions, the theory of which shows that the kappa and the entropic q indices are connected through $\kappa = 1/(q - 1)$ [4,5]. It has found application [6–10] in the physics of earthquakes and especially in the description of the asperities in the faults on which earthquakes occur. In particular, Sotolongo-Costa and Posadas (SCP) proposed [6] a model for earthquake (EQ) dynamics related to the Tsallis nonextensivity framework: It consists basically of two rough profiles interacting via fragments filling the gap between them (cf. the fragments were earlier produced by breakage of the plates). In other words, the fundamental idea of this model consists of the fact that the space between faults is filled with the residues of the breakage of the tectonic plates from where the faults

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originate. In this model, the mechanism of EQ triggering assigns an important role in the fragments which may act as roller bearings, and also as hindering entities of the relative motion of the plates. SCP applied the maximum entropy principle for the surface σ of the fragments by employing Tsallis entropy[2] S_q and obtained the fragment size distribution. By also considering a relation between the energy ϵ emitted by an EQ and the linear size $r(\sim \sqrt{\sigma})$ of the broken fragment, SCP deduced an energy distribution function for EQs that explicitly depends on q. The SCP model was revisited by Silva et al. [7] who made two key improvements: The first one was using of a different definition [11] for mean values in the context of Tsallis nonextensive statistics. The second improvement by Silva et al. refers to the introduction of a scaling law, i.e., $\epsilon \sim r^3$, between the released relative energy ϵ and the size r of fragments which substantially differs from the assumption $\epsilon \sim r$ used by SCP. These two improvements resulted in a q-dependent EQ magnitude distribution usually called generalized Gutenberg-Richter law (for details see Ref. [12]). Hence, on the basis of the observed EQ magnitude distribution one can deduce [12,13] the entropic index q and study how it varies before a strong EQ (for a review see Chapter 2 of Ref. [10], see also Refs. [14–17]). It is the basic aim of this manuscript to investigate whether this q variation can contribute to the identification of the occurrence time of an impending major EQ. In particular, we shall focus here our investigation on the Tohoku EQ that occurred on 11 March 2011 with magnitude $M9$ which is the largest magnitude EQ ever recorded in Japan.

Earthquakes exhibit complex correlations in time, space and magnitude (e.g., [18–24]). It is widely accepted [25,26] that the observed EQ scaling laws [27] indicate the existence of phenomena closely associated with the proximity of the system to a critical point. For the analysis of complex time series, among which seismicity is just an example, a new procedure termed natural time analysis was introduced in the beginning of 2000 s [28] (e.g., see Refs. [29–31] and references therein) which has the privilege to identify when the system approaches the critical point and hence estimate the occurrence time of the mainshock in the case of seismicity. Natural time analysis has found applications in diverse fields and is currently considered as the basis for a new methodology to estimate the seismic risk by Turcotte and coworkers [24,32–34] termed (seismic) Nowcasting. In Section 2 we shortly describe how this analysis can serve for the estimation of the occurrence time of an impending main shock and in Section 2 the data and analysis are presented. Our results are described in Section 4 and a summary of our main conclusions is given in Section 5.

2 Natural time analysis: background and the identification of the occurrence time of an impending mainshock

For a time series comprising N EQs, the natural time χ_k is defined as $\chi_k = k/N$, where k means the k-th EQ with energy Q_k. Thus, the raw data for our investigation, to be read from the EQ catalog, are $\chi_k = k/N$ and $p_k = Q_k/\sum_{n=1}^{N} Q_n$, where p_k is the normalized energy. In natural time, we are interested in the order and energy of events but not in the time intervals between events. Using natural time analysis our previous works [28,35] showed that this new time frame made the lead time of prediction as short as a few days as follows.

We calculate from the catalog a parameter called κ_1, defined as follows [28,35,36]

$$\kappa_1 = \sum_{k=1}^{N} p_k(\chi_k)^2 - \left(\sum_{k=1}^{N} p_k\chi_k\right)^2 = \langle\chi^2\rangle - \langle\chi\rangle^2. \quad (1)$$

We start the calculation of κ_1 (which has been shown to be order parameter of seismicity [35]) at the time of initiation of Seismic Electric Signals (SES), the transient low frequency ($\leq$ 1Hz) changes of the electric field of Earth that have long been successfully used for short-term EQ prediction [28]. The area to suffer a main shock is estimated on the basis of the selectivity map [28] of the station that recorded the corresponding SES. Thus, we now have an area in which we count the small EQs of magnitude greater than or equal to a certain magnitude threshold M_{thres} that occur after the initiation of the SES. We then form time series of seismic events in natural time for this area each time a small EQ occurs, in other words, when the number of the events increases by one. The κ_1 value for each time series is computed for the pairs (χ_k, p_k) by considering that χ_k is "rescaled" to $\chi_k = k/(N+1)$ together with rescaling $p_k = Q_k / \sum_{n=1}^{N} Q_n$ upon the occurrence of any additional event in the area. The resulting number of thus computed κ_1 values is usually of the order 10^2 to 10^3 depending, of course, on the magnitude threshold adopted for the events that occurred after the SES initiation until the main shock occurrence. When we followed this procedure, it was found empirically that the values of κ_1 converge to 0.070 a few days before main shocks. Thus, by using the date of convergence to 0.070 for prediction, the lead times, which were a few months to a few weeks or so by SES data alone, were made, although empirically, as short as a few days [35,37] (the condition $\kappa_1 = 0.070$ has been later [28] shown theoretically as well). For example, the prominent seismic swarm activity in 2000 in the Izu Island region, Japan, was preceded by a pronounced SES activity 2 months before it, and the approach of κ_1 to 0.070 was found a few days before the swarm onset [38].

When SES data are not available, we rely on the following two recent findings obtained on the basis of natural time analysis: First, the fluctuations β of the order parameter κ_1 of seismicity in a large area exhibit a minimum a few months before a major EQ almost simultaneously with the initiation of an SES activity [39]. Second, a spatiotemporal study of this minimum unveils an estimate of the epicentral area of the impending major EQ [40]. The application of this procedure to the determination of the occurrence time of the M9 Tohoku EQ in reference [41] was made as follows.

Concerning the starting time of the natural time analysis of seismicity, the date of 5 January 2011 was chosen since it is the date of the appearance of the minimum of the fluctuations of the order parameter of seismicity before this major EQ. This, which remarkably is the deepest minimum observed during the period 1984–2011 investigated, almost coincides with the initiation of an SES activity since anomalous magnetic field variations appeared in the Z component during the period 4–14 January 2011 at measuring sites lying at epicentral distances of around 130 km [42–44]. As for the estimation of the epicentral location of the impending mainshock without making use of SES data, this has been achieved [40] as follows: By dividing the entire Japanese region $N_{25}^{46}E_{125}^{148}$ into small areas, a calculation of the fluctuations of κ_1 of seismicity is carried out on them. Some small areas show a minimum of the fluctuations almost simultaneously with the minimum in the entire Japanese region (on 5 January 2011) and such small areas cluster within a few hundred km from the actual epicenter, thus leading to an estimate of the candidate epicentral area. A computation of the κ_1 values of seismicity in that area was made by starting from 5 January 2011. The computed κ_1 values clearly showed that the condition $\kappa_1 = 0.070$ was not satisfied for all magnitude thresholds at least until the M7.3 foreshock at 11:45 LT on 9 March 2011. Hence, here we plot in expanded time scale in Figure 1a the κ_1 values of seismicity from 00:00 LT on 9 March 2011 until the Tohoku EQ occurrence which reveal that the condition $\kappa_1 = 0.070$ is fulfilled for $M_{thres} = 4.2$–5.0 in the morning of 10 March 2011 upon the occurrence of the EQs from 08:36 to 13:14 LT, i.e., almost one day before the Tohoku EQ, see the gray shaded area in Figure 1a. This signals

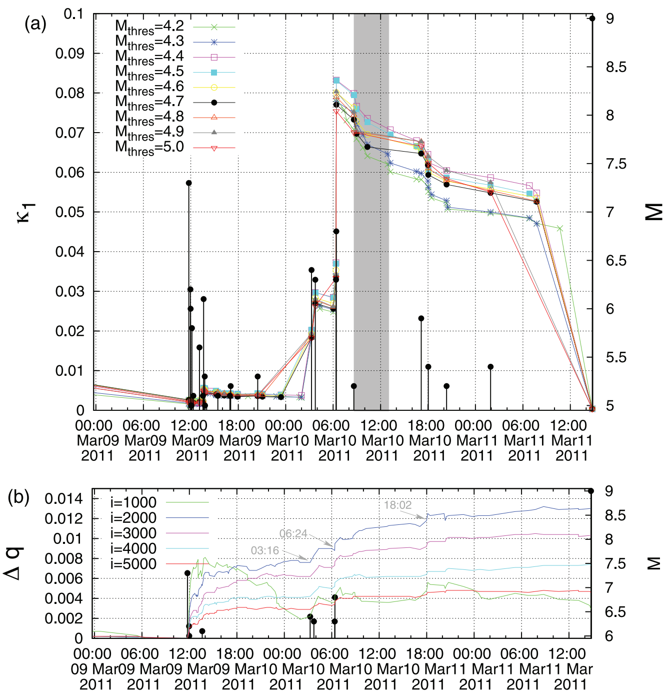

Fig. 1. (a) The κ_1 values of seismicity versus the conventional time since 00:00 LT on 9 March 2011 until the $M9$ Tohoku EQ occurrence. The shaded area marks the period from 08:36 to 13:14 LT on 10 March 2011, almost one day before the $M9$ Tohoku EQ occurrence, during which the condition $\kappa_1 = 0.070$ is fulfilled exhibiting magnitude threshold invariance for $M_{thres} = 4.2$–5. In panel (b), we depict the change Δq calculated by subtracting from each current q-value the one calculated just before the occurrence of the $M7.3$ foreshock at 11:45LT on 9 March 2011. The latter q-values, for each scale i, can be visualized in Figure 2a.

that the mainshock was going to occur within the next few days or so as actually happened.

3 Data and analysis

We used the Japan Meteorological Agency (JMA) seismic catalogue, e.g., see references [39,40], and consider all EQs of magnitude M$\geq$ 3.5 from 1984 until the Tohoku EQ occurrence on 11 March 2011 within the area 25°−46°N, 125°−148°E. The energy of EQs was obtained from the JMA magnitude M after converting [45] to the moment magnitude M_w [46]. Setting a threshold $M = 3.5$ to assure data completeness, there exist 47,204 EQs in the area under discussion. Thus, we have on the average ∼150 EQs per month for the area considered. Since SES activities [36] exhibiting critical behavior [28] have lead times ranging from a few weeks up to around $5\frac{1}{2}$ months [28] and in view of the fact that a significant change in the temporal correlations between EQ magnitudes occurs when comparing the two stages that correspond to the periods before and just after the initiation of an SES activity [47] we start our investigation from the scale of $i \sim 10^3$ events, which is of the order of the number of seismic events $M \geq 3.5$ that occur during a period around the maximum lead time of SES activities.

4 Results

We first investigate whether the behavior in Figure 1a is accompanied by noticeable variations of Tsallis entropic index q. Thus, we plot in Figure 1b the Δq values for various scales versus the conventional time during the same period as in Figure 1a, i.e., from 00:00 LT on 9 March 2011 until the $M9$ Tohoku EQ occurrence. A close inspection of these plots in Figure 1b reveals that just after this $M7.3$ foreshock, i.e., at around 03:16 UT on 10 March 2011 as well as at 06:24 LT on 10 March 2011, i.e., upon the occurrence of the EQs with $M6.4$ and $M6.8$, respectively, steep increases of the Δq value are observed. Interestingly, these two Δq changes occur around a couple of hours before the fulfillment of the condition κ_1= 0.070 deduced from natural time analysis in the gray region of Figure 1. These are marked with arrows in Figure 1b. A third Δq increase is also observed at 18:02 LT just after the fulfillment of the condition $\kappa_1 = 0.070$.

The q value for several scales, i.e., i =1000, 2000, 3000, 4000 and 5000 events $M \geq 3.5$, versus the conventional time during the almost 27 year period from 1 January 1984 until the $M9$ Tohoku EQ occurrence on 11 March 2011 has been plotted in Figure 6 of reference [30]. Here, in Figure 2a we restrict ourselves to an almost three month excerpt showing what happens just before this $M9$ EQ occurrence. An inspection of Figure 2a shows that there exist two noticeable changes of the q value: First, a prominent increase upon the occurrence of the $M7.8$ EQ on 22 December 2010 almost coinciding with the minimum of the entropy change ΔS upon time reversal [29] (for ΔS definition see Ref. [48]). Second, a smaller but evident increase upon the occurrence of the foreshock $M7.3$ occurrence on 9 March 2011. The former change of the q value has been studied in detail in references [30,31] and the following conclusions have been drawn: Upon applying natural time analysis to the Japanese seismic data, it was found that it is accompanied by an abrupt increase of the fluctuations of the order parameter of seismicity exhibiting a functional form discussed [49] by Penrose and coworkers in computer simulations of phase transition kinetics using the ideas of Lifshitz and Slyozov (LS) [50]. In what remains, we focus on the study of the latter increase of q value observed upon the occurrence of the $M7.3$ foreshock on 9 March 2011. To better visualize the details of this increase, we plot in Figure 2b for all the aforementioned scales i = 1000–5000 events the Δq values versus the conventional time during the almost two day period from 11:54 on 9 March 2011 until the $M9$ Tohoku EQ occurrence. Furthermore, in Figure 2c we give the log-log plot of Δq versus the elapsed time $(t - t_0)$ in days since the establishment of scaling behavior after the occurrence of the $M7.3$ foreshock on 9 March 2011 to which we now turn. In particular, we investigate whether it obeys a scaling behavior of the form

$$\Delta q = A(t - t_0)^c \tag{2}$$

according to the seminal work by Lifshitz and Slyozov [50] and independently by Wagner [51] (LSW theory) on phase transitions which shows that the time growth of the characteristic size of the minority phase droplets grows with time as $t^{1/3}$, i.e., the exponent c has a value very close to 1/3. From the slope of the plot in Figure 2c we find $c \approx 1/3$ which conforms with LSW theory. Further, the scaling behavior becomes evident from Figure 3 which shows that the inverse of the prefactor A in equation (2) varies almost linearly with the scale i used for the calculation of seismicity either after the occurrence of the $M7.8$ EQ on 22 December 2010 (red) or after the $M7.3$ foreshock on 9 March (green) visualized in Figure 2a. In particular, we find $A \approx 36/i$ and $A \approx 26/i$ for these two cases of Δq changes after the $M7.8$ and $M7.3$ EQs, respectively. The fact that the scaling behavior of equation (2) is obeyed here, could be understood in the following context, if we also recall that an EQ occurrence can be

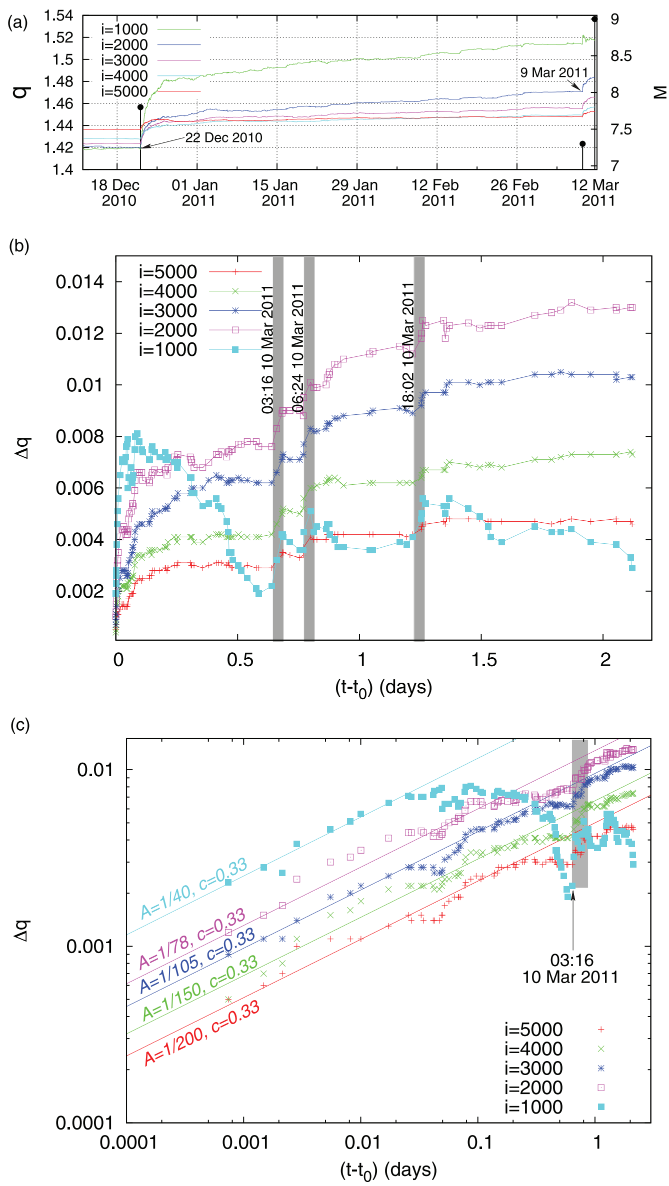

Fig. 2. In (a) the q values for various scales $i = 1000$–5000 events of seismicity is plotted versus the conventional time for an almost three months period before the $M9$ Tohoku Eq occurrence. In (b) and (c) the change Δq is plotted versus the conventional time elapsed $(t - t_0)$ in days approximately 9 minutes after the $M7.3$ foreshock occurrence during the almost two day period in lin–lin and log–log, respectively.

considered as phase transition: LS, in their classic paper [50], studied the kinetics of phase transitions when considering diffusion-limited growth of grains in supersatured solid solutions (for the basic ideas of the thermodynamic theory of nucleus formation in a phase transition see p. 427 of Ref. [52] in conjunction with paragraph 162 of Ref. [53]). LS showed that asymptotically the distribution over sizes tends to a self-similar universal shape, while the critical, average, and maximum sizes *all* change as

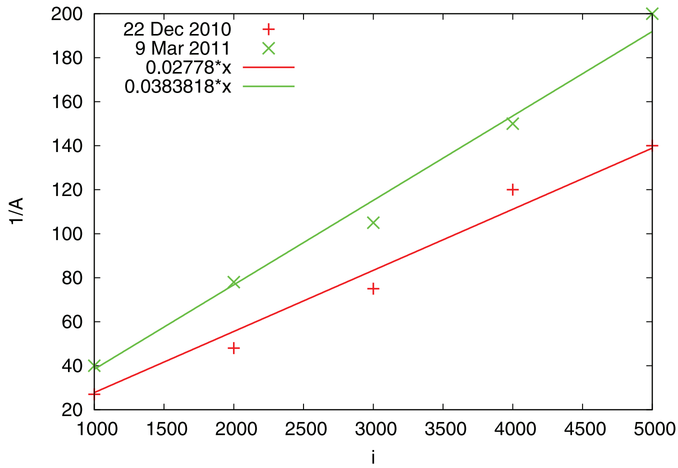

Fig. 3. The inverse of the prefactor A in equation (2) as it results from Figure 8 of reference [30] and from Figure 2c as a function of the scale i used in the calculation of seismicity after the occurrence of the $M7.8$ EQ on 22 December 2010 (red) and the $M7.3$ foreshock on 9 March 2011 (green), respectively.

a cubic root of time, i.e., the $t^{1/3}$ law. It was further demonstrated that these results were extremely robust, and remained valid even if elastic stress, anisotropy, and other effects were taken into consideration.

5 Summary and main conclusions

Almost a day before the $M9$ Tohoku EQ occurrence on 11 March 2011, natural time analysis revealed that the order parameter κ_1 of seismicity, and in particular from 08:36 to 13:14 LT on 10 March 2011, fulfilled the critical condition $\kappa_1 = 0.070$ which signals that the main shock is going to occur within the next few days or so. Just before this period, the following two important findings emerged here: First, the Tsallis entropic index q showed distinct changes at 03:16 LT and 06:24 LT on 10 March 2011. Second, upon the occurrence of the $M7.3$ foreshock on 9 March 2011,a prominent increase of the Tsallis entropic index q was observed that exhibited a scaling behavior with a characteristic exponent $1/3$ which conforms to the seminal work by Lifshitz-Slyozov and independently by Wagner on phase transitions predicting that the time growth of minority phase droplets grows with time t as $t^{1/3}$. As for the prefactor A in equation (2) of LSW theory, we find that it increases when the scale i decreases, in contrast to the complexity measure Λ quantifying the fluctuations of the entropy change under time-reversal for which the LSW prefactor A increases upon increasing the scale i [30].

Author contribution statement

E.S.S., N.V.S. and P.A.V. designed research. E.S.S and N.V.S. performed the calculations. E.S.S, N.V.S. and P.A.V. wrote the paper.

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