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Research paper

Natural time analysis together with non-extensive statistical mechanics shorten the time window of the impending 2011 Tohoku $M9$ earthquake in Japan[☆]

P.A. Varotsos [a,*], N.V. Sarlis [a], E.S. Skordas [a], Toshiyasu Nagao [b], Masashi Kamogawa [b]

[a] Physics Department, National and Kapodistrian University of Athens, Section of Condensed Matter Physics and Solid Earth Physics Institute, Panepistimiopolis, Zografos, Athens, 15784, Greece
[b] Natural Disaster Research Section (NaDiR), Global Center for Asian and Regional Research, University of Shizuoka, 3-6-1, Takajo, Aoi-Ku, Shizuoka-City, Shizuoka, 420-0839, Japan

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ABSTRACT

Applying natural time analysis (NTA) to the Japanese seismic data, we have found that the system enters the critical stage upon the occurrence of M=4.2-5.0 earthquakes from 08:36 to 13:14 local time (LT) on 10 March 2011, i.e., almost one day before the $M9$ Tohoku earthquake on 11 March 2011. In addition, here, we find that just after this period the entropy change ΔS under time reversal of NTA along with the Tsallis entropy of non-extensive statistical mechanics (NESM) show distinct simultaneous changes. This simultaneous appearance enables the shortening of the time window of the impending mainshock to several hours. Furthermore, upon the occurrence of the $M7.3$ foreshock at 11:45 LT on 9 March 2011, the following fact emerged: The Tsallis entropy of NESM exhibited a scaling behavior with a characteristic exponent 1/3 that conforms to Lifshitz–Slyozov–Wagner theory for phase transitions as had been also observed in NTA for the fluctuations of the entropy change ΔS under time reversal upon a $M7.8$ earthquake occurrence in Japan on 22 December 2010. The latter obeyed the form $A(t-t_0)^c$ where c is approximately equal to 1/3 and the pre-factors A are proportional to the scale i (number of events) while t_0 is almost 0.2 days after this $M7.8$ earthquake.

1. Introduction

Earthquake (EQ)[1] occurrences exhibit complex correlations in time, space and magnitude (e.g., [1–6]). It is widely accepted [7–9] that the observed EQ scaling laws [10] indicate the existence of phenomena closely associated with the proximity of the system to a critical point. To analyze complex time series, among which seismicity is just an example, natural time analysis (NTA) was introduced in the beginning of 2000s (e.g., see Ref. [11–13] and references therein) which enables recognition of when the system enters the critical stage [9,14]. After this recognition, it has been found in the

☆ To the memory of the Academician Seiya Uyeda.
* Corresponding author.
E-mail addresses: pvaro@otenet.gr (P.A. Varotsos), nsarlis@phys.uoa.gr (N.V. Sarlis), eskordas@phys.uoa.gr (E.S. Skordas), nagao@scc.u-tokai.ac.jp (T. Nagao), kamogawa@u-shizuoka-ken.ac.jp (M. Kamogawa).
1 Abbreviations: EQ = Earthquake, GR = Gutenberg–Richter, JMA = Japan Meteorological Agency, LT = Local time, LSW = Lifshitz–Slyozov–Wagner, NESM = non-extensive statistical mechanics, NTA = Natural time analysis, SES = Seismic electric signals.

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case of seismicity that the mainshock (critical point) occurs a few days to around one week later [14]. It is the objective of the present work to shorten this time window of one week or so, aiming to identify more accurately the occurrence time of the mainshock (within a day or around several hours). To achieve this goal in the case of the Tohoku mega-EQ that occurred on 11 March 2011, we also employ entropic measures and in particular the entropy change ΔS in NTA under time reversal as well as the Tsallis entropy (see below). NTA has found applications in diverse fields compiled in Ref. [9] and is currently considered as the basis for a new procedure to estimate the seismic risk by Turcotte, Rundle and coworkers [15–18] termed earthquake nowcasting. NTA will be shortly presented in the next Section together with an application on how this analysis can serve for the estimation of the occurrence time of an impending mainshock. In Section 3 the data and the analysis are presented and our results are given in Section 4. Finally, in Section 5, we summarize our main conclusions.

Non-extensive statistical mechanics (NESM) [19], pioneered by Tsallis [20,21], provides a framework for the study of complex systems in their non-equilibrium stationary states, as well as in systems with (multi)fractal and self-similar structures, long-range interacting systems etc., resulting in power-law asymptotic behavior frequently observed in nature. It is a generalization of the classical statistical theory of Boltzmann and Gibbs and the involved entropy is a mono-parametrical function of the probability distribution. The entropic parameter q is incorporated in the expression of Tsallis entropy, S_q, given by

$$S_q = \frac{k_B}{q-1}\left(1 - \sum_{i=1}^{\infty} p_i^q\right) \tag{1}$$

in terms of the probability distribution, and can attain any value, while for $q \to 1$, recovers the Boltzmanian entropy and the Boltzmann–Gibbs statistical mechanics (cf. k_B stands for the Boltzmann constant).

Tsallis [19] outlined various disadvantages of Boltzmann–Gibbs entropy (S_{BG}) and proposed the aforementioned definition of non-additive entropy. Tsallis entropy satisfies:

$$\frac{S_q(A+B)}{k_B} = \frac{S_q(A)}{k_B} + \frac{S_q(B)}{k_B} + (1-q)\frac{S_q(A)}{k_B}\frac{S_q(B)}{k_B}, \tag{2}$$

where A and B are two different systems and the concepts of superextensivity, extensivity, and subextensivity correspond to $q < 1$, $q = 1$, and $q > 1$, respectively (see also below on this point). We strongly recommend to the reader to go through the recent paper by Tsallis – see Ref. [22] – which removes a lot of misunderstandings that led to an unjustified criticism of NESM. For example, concerning the point "Additivity versus Extensivity", Tsallis [22] explained that some people do not distinguish – clearly enough, and even at all – the concepts of "extensivity" and "additivity", applicable to both entropy and energy. This is quite unfortunate since this distinction ought to be made in any introductory talk on the subject. Indeed, it plays a foundational role in non-extensive statistical mechanics. Let us address now these two important notions, focusing specifically on entropic additivity and entropic extensivity. Following Penrose [23], an entropic functional $S(\{p_i\})$ is said "additive" if, for two *probabilistically independent* systems A and B (i.e., $p_{ij}^{A+B} = p_i^A p_j^B$), one verifies $S(A+B) = S(A) + S(B)$, in other words, if

$$S(\{p_i^A p_j^B\}) = S(\{p_i^A\}) + S(\{p_j^B\}) \tag{3}$$

is verified. Otherwise, $S(\{p_i\})$ is said "nonadditive". It immediately follows that $S_{BG} = -k_B \sum_{i=1}^{N} p_i \ln p_i$ is additive. In contrast, S_q satisfies Eq. (2), and hence

$$S_q(A+B) = S_q(A) + S_q(B) + \frac{1-q}{k_B} S_q(A) S_q(B). \tag{4}$$

Therefore, unless $(1-q)/k_B \to 0$, S_q is nonadditive. Let us now address the other important entropic concept, namely, extensivity. An entropy $S(N)$ is said "extensive" if a specific entropic functional is applied to a specific class of many-body systems with $N = L^d$ particles, where L is its dimensionless linear size and d its spatial dimension, and satisfies the thermodynamical expectation

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty, \tag{5}$$

hence, $S(N) \propto N$ for $N \gg 1$. Therefore, entropic additivity only depends on the entropic functional, whereas entropic extensivity depends on *both* the chosen entropic functional *and* the system itself (i.e., its constituents and the correlations among them). To illustrate this fundamental distinction Tsallis presented four (see Fig. 1 of Ref. [22]), among infinitely many, equal-probability typical examples of $W(N)$ ($N \to \infty$), where W is the total number of possibilities whose probability does not vanish. Note also that recently the fundamental concept of entropy defect [24,25] has been introduced to account for the non-additivity of Tsallis entropy.

NESM is the background of kappa distributions, the theory of which shows that the kappa and the entropic q indices are connected through $\kappa = 1/(q-1)$ [26,27]. It has found application [28–35] in the physics of earthquakes and especially in the description of the asperities in the faults on which earthquakes occur. In particular, Sotolongo-Costa and Posadas [28] proposed a model for EQ dynamics related to the Tsallis nonextensivity framework: It consists basically of two rough profiles interacting via fragments filling the gap between them (cf. the fragments were earlier produced by breakage of the plates). In this model, the released seismic energy ϵ is related to the size of the fragments that fill the space between fault blocks. Silva et al. [29] slightly revised the fragment–asperity model using a volumetric relationship between seismic energy and fragment size instead of a linear one, in accordance with the standard theory of seismic moment scaling with rupture length [36]. Subsequently, Darooneh and Mehri [37] and Telesca [38,39] further refined the fragment–asperity model by introducing a function between EQ magnitude (M) and relative energy (ϵ) released as follows [40]:

$$M \propto \frac{2}{3}\log_{10}(\epsilon) \tag{6}$$

These studies finally lead [41] to a generalized Gutenberg–Richter (GR) relationship which results in

$$b = 2\left(\frac{2-q}{q-1}\right) \tag{7}$$

for the b-value of the conventional GR law [42] (cf. the latter states that the number $N(>M)$ of EQs above a certain magnitude M in a given area and for a given time period scales as $N(>M) \propto 10^{-bM}$), where the q values obtained from different regions of the world [41] are all $q \approx 1.5-1.7$. In a very recent work [43], Posadas and Sotolongo-Costa obtained the relation:

$$S_q = \frac{1-\int_0^\infty p^q(\sigma)d\sigma}{q-1} = \frac{1-(2-q)^{\frac{1}{2-q}}}{q-1} \tag{8}$$

This equation allows us to find the value of the entropy for a dataset and to study its behavior as a function of the non-extensivity q parameter; therefore, if a windowing process is carried out (i.e., choosing a certain number i of earthquakes and sliding the window in time), it is possible to visualize the dynamic evolution of the seismic series in terms of the non-additive entropy.

2. Background of natural time analysis and the estimation of the occurrence time of an impending mainshock

For a series of N events, which actually is a temporal point pattern, see, e.g., Ref. [44], we define as natural time χ_k for the occurrence of the kth event the quantity $\chi_k = k/N$ [11–13]. In doing so, we ignore the time intervals between consecutive events, but preserve their order and energy Q_k. NTA is carried out by studying the evolution of the pair (χ_k, p_k), where the quantity $p_k = Q_k/\sum_{n=1}^{N} Q_n$ is the normalized energy for the kth event, and using the normalized power spectrum $\Pi(\omega) \equiv |\Phi(\omega)|^2$ (cf. ω stands for the angular natural frequency) defined by $\Phi(\omega) = \sum_{k=1}^{N} p_k \exp(i\omega\chi_k)$. $\Phi(\omega)$ is the characteristic function of p_k for all $\omega \in \mathcal{R}$, since p_k can be regarded as a probability for the occurrence of the kth event at χ_k. In NTA, the behavior of $\Phi(\omega)$ is studied at $\omega \to 0$, because all the moments of the distribution of p_k can be estimated from the derivatives $d^m\Phi(\omega)/d\omega^m$ (for m positive integer) of the characteristic function $\Phi(\omega)$ at $\omega \to 0$. For this purpose, a quantity κ_1 was defined from the Taylor expansion $\Pi(\omega) = 1 - \kappa_1\omega^2 + \kappa_2\omega^4 + \cdots$ where

$$\kappa_1 = \langle\chi^2\rangle - \langle\chi\rangle^2 = \sum_{k=1}^{N} p_k(\chi_k)^2 - \left(\sum_{k=1}^{N} p_k\chi_k\right)^2. \tag{9}$$

The quantity κ_1 becomes equal to 0.070 at the critical state for a variety of dynamical systems [9,14,45–47]. In general, this quantity is useful in identifying the approach to a critical point. In NTA of the seismicity, a careful inspection reveals [48] that the quantity κ_1 may be considered as an order parameter of seismicity, see also Refs. [49,50].

In the 1980s, a criticality physical model [51] inspired the study of transient electric signals before major EQs [52]. A series of such transient changes of the Earth's electric field are termed Seismic Electric Signals (SES) activities [53], the study of the physical properties of which reveals the magnitude and the epicentral area of an impending EQ.

In 2001, the identification of the occurrence time of the impending mainshock was made as follows [11]: Just after a SES activity initiation, we form time series of seismic events in natural time for the area that constitutes the selectivity [52,53] map (of the geoelectrical station that recorded the SES activity) each time a small EQ of energy Q_k occurs, in other words, when the number of the events increases by one. The κ_1 value for each time series is computed for the pairs (χ_k, p_k) by considering that χ_k is "rescaled" to $\chi_k = k/(N+1)$ together with rescaling $p_k = Q_k/\sum_{n=1}^{N+1} Q_n$ upon the occurrence of any additional event in the area. When we followed this procedure, it was found empirically that the values of κ_1 converge to 0.070 usually a few days before mainshocks. Thus, by using the date of convergence to 0.070 for prediction, the lead times, which were a few months to a few weeks or so by SES data alone, were made, although empirically, as short as a few days [54–56] up to maximum of 11 days or so (cf. a theoretical explanation of the convergence $\kappa_1 = 0.070$ for prediction has been later achieved [14]). For example, the prominent seismic swarm activity in 2000 in the Izu Island region, Japan, was preceded by a pronounced SES activity 2 months before and the approach of κ_1 to 0.070 was found a few days before the swarm onset [57].

However, when SES data are not available, which is usually the case, it is not possible to follow the above procedure. This is so because neither the initiation time of the SES activity, nor the candidate area (i.e., the SES selectivity map) will be available. However, both these difficulties will be overcome as follows: In particular, we showed -see below- that the initiation of a SES activity is almost simultaneous with the minimum [58] of the fluctuations of the order parameter of seismicity κ_1, hence the latter can serve for the information of the former. Furthermore, we have found [59] that the epicenter of an impending major EQ can be estimated by means of the study of the spatiotemporal variations of the order parameter of seismicity, hence the second difficulty can also be overcome (see also the schematic diagram in Ref. [60]). As an example to overcome both these difficulties we have presented [61] the case of the identification of the occurrence time of the impending $M9$ Tohoku mega-EQ that occurred on 11 March 2011 in Japan. Note that an alternative procedure for the identification of the occurrence time of the Tohoku $M9$ EQ in 2011, has been presented in Ref. [62] by employing the fluctuations of the seismicity entropy change under time reversal quantified by a complexity measure (Λ_i) defined below.

The entropy S in natural time (which is a *dynamic* entropy and not a static, e.g., Shannon entropy [63–65]) defined in Ref. [66] is

$$S = \langle \chi \ln \chi \rangle - \langle \chi \rangle \ln \langle \chi \rangle, \tag{10}$$

where $\langle f(\chi) \rangle = \sum_{k=1}^{N} p_k f(\chi_k)$ denotes the average value of $f(\chi)$ weighted by p_k, i.e., $\langle \chi \ln \chi \rangle = \sum_{k=1}^{N} p_k (k/N) \ln(k/N)$ and $\langle \chi \rangle = \sum_{k=1}^{N} p_k (k/N)$. The entropy obtained by Eq. (10) upon considering [9,67] the time-reversal $\hat{T}$, i.e., $\hat{T} p_k = p_{N-k+1}$, is labeled by S_-, i.e.,

$$S_- = \sum_{k=1}^{N} p_{N-k+1} \frac{k}{N} \ln\left(\frac{k}{N}\right) - \left(\sum_{k=1}^{N} p_{N-k+1} \frac{k}{N}\right) \ln\left(\sum_{k=1}^{N} p_{N-k+1} \frac{k}{N}\right). \tag{11}$$

S_- is different from S, thus there exists a change $\Delta S \equiv S - S_-$ in natural time under time reversal, as already mentioned in the Introduction. Hence, S does satisfy the condition to be time-reversal asymmetric [9,65,67].

The quantity ΔS is of key importance to identify also when a system approaches a dynamic phase transition. Its calculation is carried out by means of a window of length i (= number of successive events), sliding each time by one event, through the whole time series. The entropies S_i and $(S_-)_i$, and therefrom their difference ΔS_i, are calculated each time. Thus, we form a new time series comprising successive ΔS_i values.

Computing the standard deviation $\sigma(\Delta S_i)$ of the time series of $\Delta S_i \equiv S_i - (S_-)_i$, the complexity measure Λ_i, which is particularly very useful for the analysis of EQ catalogs, is defined by [9,68]

$$\Lambda_i = \frac{\sigma(\Delta S_i)}{\sigma(\Delta S_{100})} \tag{12}$$

where the denominator stands for the standard deviation $\sigma(\Delta S_{100})$ of the time series of ΔS_i of $i = 100$ events. Note that the selection of a different scale in the denominator, e.g., $i = 50$ or 200 events, instead of $i = 100$ events, would change of course the numerical values obtained but the whole behavior and physical picture of the results concerning the time evolution of Λ_i would remain the same [69]. Λ_i quantifies how the statistics of ΔS_i time series varies upon changing the scale from 100 to another scale i, and is of profound importance to study the dynamical evolution of a complex system (see p. 159 of Ref. [9]).

Natural time analysis reveals that the entropy change under time reversal is minimized before a major EQ. This is consistent with the fact that upon analyzing the Olami–Feder–Christensen (OFC) model for EQs in natural time, a non-zero change ΔS of the entropy in natural time upon time reversal is identified [9,70], which reveals a breaking of the time symmetry, thus reflecting the predictability in the OFC model, see, e.g., [71]. In particular, in the OFC model, it was found (see Fig. 8.12, p.361 of Ref. [9]) that the value of ΔS_i exhibits a clear minimum [9] (or maximum if we define as in Ref. [70] $\Delta S \equiv S_- - S$, instead of $\Delta S \equiv S - S_-$) before large avalanches. In view of this finding, natural time analysis of all EQs in Japan (see Fig. 1) above a magnitude threshold (M_{thres}) from 1 January 1984 until the occurrence of the super-giant $M9$ Tohoku EQ on 11 March 2011 was made [72]. It was found that for longer scales, i.e, $i > 3500$ events, the minimum of ΔS is observed on 22 December 2010. This is consistent with the aforementioned finding in the OFC model. In addition, studying the complexity measure Λ_i versus i at the scales $i = 2000$, 3000 and 4000 events, Varotsos et al. [73] found an evident increase $\Delta \Lambda_i$ on 22 December 2010 upon the occurrence of a $M7.8$ EQ obeying a scaling behavior of the form $\Delta \Lambda_i = A(t - t_0)^c$, where the exponent c is independent of i with a value very close to 1/3, while the pre-factors A are proportional to i and t_0 is approximately 0.2 days after the $M7.8$ EQ occurrence. This behavior conforms to the seminal work by Lifshitz and Slyozov [74] and independently by Wagner [75] on phase transitions which shows that the time growth of the characteristic size of the minority phase droplets grows with time as $t^{1/3}$.

3. Data and analysis

The seismic catalog of the Japan Meteorological Agency (JMA) was used as in Refs. [58,59,76]. We considered all EQs of magnitude $M \geq 3.5$ from 1984 until the Tohoku EQ occurrence on 11 March 2011 within the area 25°−46°N, 125°−148°E, see Fig. 1(a). The energy of EQs was obtained from the JMA magnitude M after converting [77] to the moment magnitude M_w [40]. Setting a threshold $M_{thres} = 3.5$ to assure data completeness, there exist 47,204 EQs in the area under discussion. Thus, we have on the average ∼150 EQs per month for the area considered.

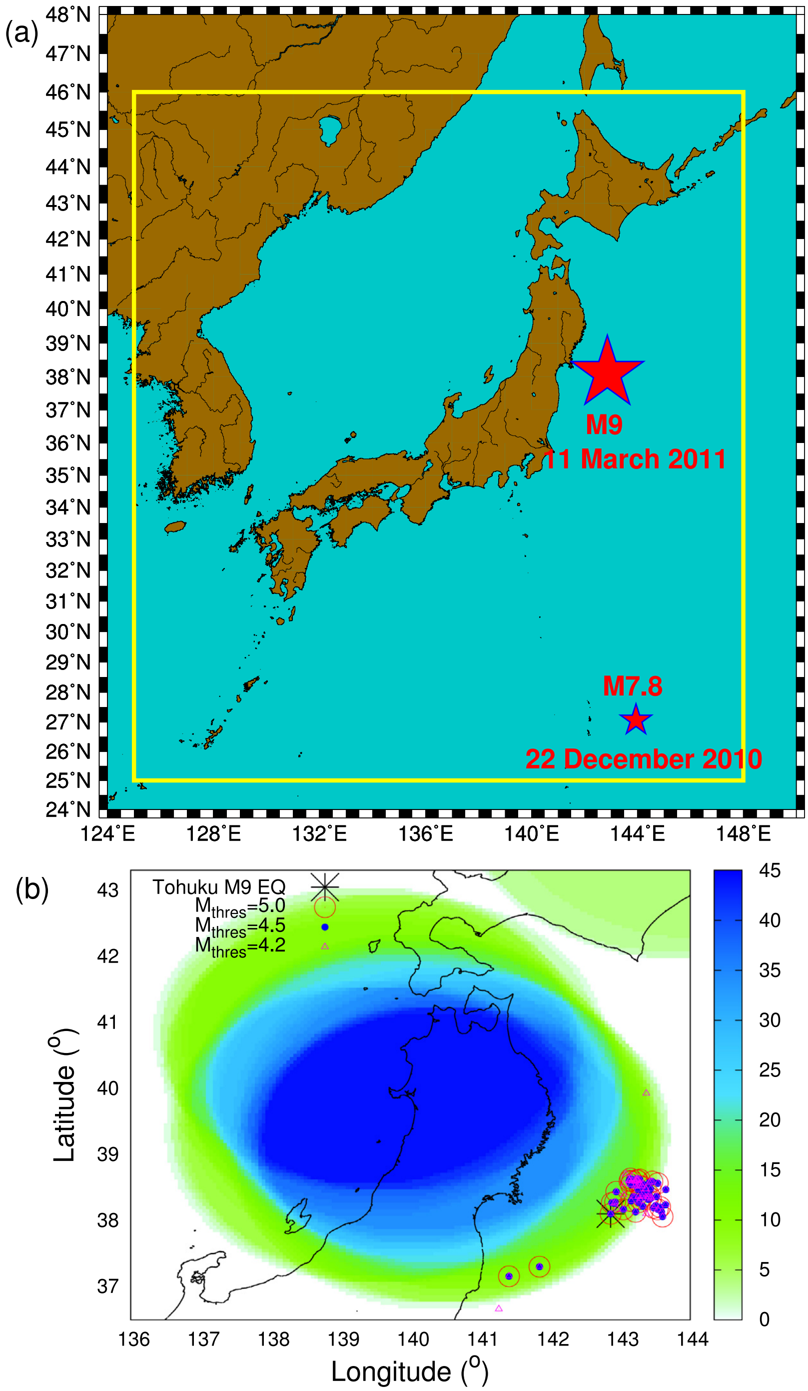

Fig. 1. Map of the entire Japanese area (a), along with the candidate epicentral area (b) for the 2011 Tohoku $M9$ EQ.

The time evolution of the entropy change ΔS under time reversal as well as the complexity measure Λ_i in NTA are studied for a number of scales i of the seismicity with $M \geq 3.5$ occurring in the whole area of Japan during the aforementioned 27 year period by choosing proper scales i as follows: We consider that investigations by means of NTA showed that there exists the following interconnection between SES activities and seismicity [78]: The fluctuations of the

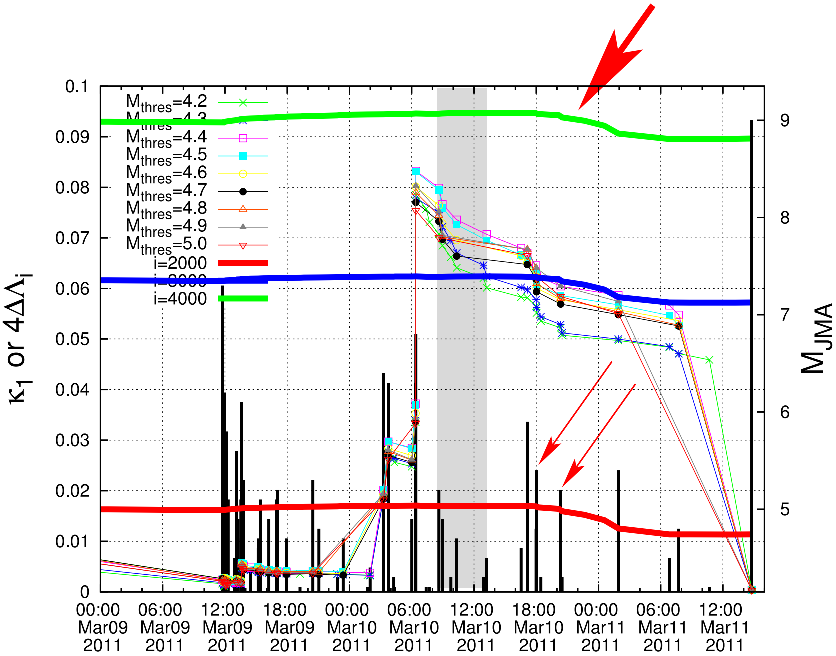

Fig. 2. The κ_1 values as well as the values of the change $\Delta\Lambda_{2000}$, $\Delta\Lambda_{3000}$ and $\Delta\Lambda_{4000}$ of the complexity measures Λ_i for $i = 2000, 3000$ and 4000 events, respectively, versus the conventional time since 00:00 LT on 9 March 2011 until the M9 Tohoku EQ occurrence. The shaded area marks the period in the morning of 10 March 2011 during which the condition $\kappa_1 = 0.070$ is fulfilled. The two thin arrows show the two EQs of magnitude 5.4 and 5.2 upon the occurrence of which simultaneous changes of ΔS_i and $\Delta S(q)$ appeared in Fig. 4. The thick arrow indicates the period of an evident decrease of the three complexity measures $\Delta\Lambda_{2000}$, $\Delta\Lambda_{3000}$ and $\Delta\Lambda_{4000}$ around 00:00LT on 11 March 2011.

order parameter κ_1 of seismicity exhibit a minimum labeled β_{min} when we observe the initiation of SES activities [53,79,80] exhibiting critical behavior [12,66,81]. The latter have lead times ranging from a few weeks up to around $5\frac{1}{2}$ months [9]. In addition, beyond this simultaneous appearance of two different geophysical observables, i.e., SES activity and seismicity, Varotsos et al. [78] showed that these two phenomena are also linked closely in space, which opened the window for a reliable estimation of the epicentral area of an impending major EQ. This has been subsequently confirmed in Ref. [59] for all major mainshocks of magnitude 7.6 or larger that occurred in Japan during 1984–2011 including the case of the M9 Tohoku EQ, see Fig. 1(b). Concerning the latter EQ, before the initiation of the SES activity, and hence before β_{min}, a stage (around 22 December 2010) has been detected in which the temporal correlations between EQ magnitudes exhibit an anticorrelated behavior [82] (since the Detrended Fluctuation [83,84] exponent α was found $\alpha = 0.35$) while after the SES activity initiation long range correlations prevail between EQ magnitudes. Thus, a significant change in the temporal correlations between EQ magnitudes occurs when comparing the two stages that correspond to the periods before and just after the initiation of a SES activity. Since this change may be captured by the time evolution of ΔS_i, we start our investigation of ΔS_i from the scale of $i \sim 10^3$ events, which is of the order of the number of seismic events $M \geq 3.5$ that occur during a period around the maximum lead time of SES activities.

4. Results

As mentioned in the Introduction, we focus on the case of the M9 Tohoku EQ occurrence on 11 March 2011. We recall that in Ref. [85] the following results have been found: almost a day before this EQ, natural time analysis revealed that the order parameter κ_1 of seismicity, and in particular from 08:36 to 13:14 LT on 10 March 2011, fulfilled [61] the critical condition $\kappa_1 = 0.070$ which signals that the system enters the critical stage and the main shock is going to occur within the next few days or so, see Fig. 2. Just before this period, the following two important findings have been observed [85]: First, the Tsallis entropic index q showed distinct changes at 03:16 LT and 06:24 LT on 10 March 2011. Second, upon the occurrence of the M7.3 foreshock on 9 March 2011, a prominent increase of the Tsallis entropic index q was observed that exhibited a scaling behavior with a characteristic exponent 1/3 which conforms to the seminal work by Lifshitz–Slyozov and independently by Wagner (LSW) on phase transitions predicting that the time growth of minority phase droplets grows with time t as $t^{1/3}$. As for the prefactor A in the quantity $A(t - t_0)^c$ of Ref. [85] of LSW theory, we find that it increases when the scale i decreases, see Fig. 3 of Ref. [85], in contrast to the complexity measure Λ_i quantifying the

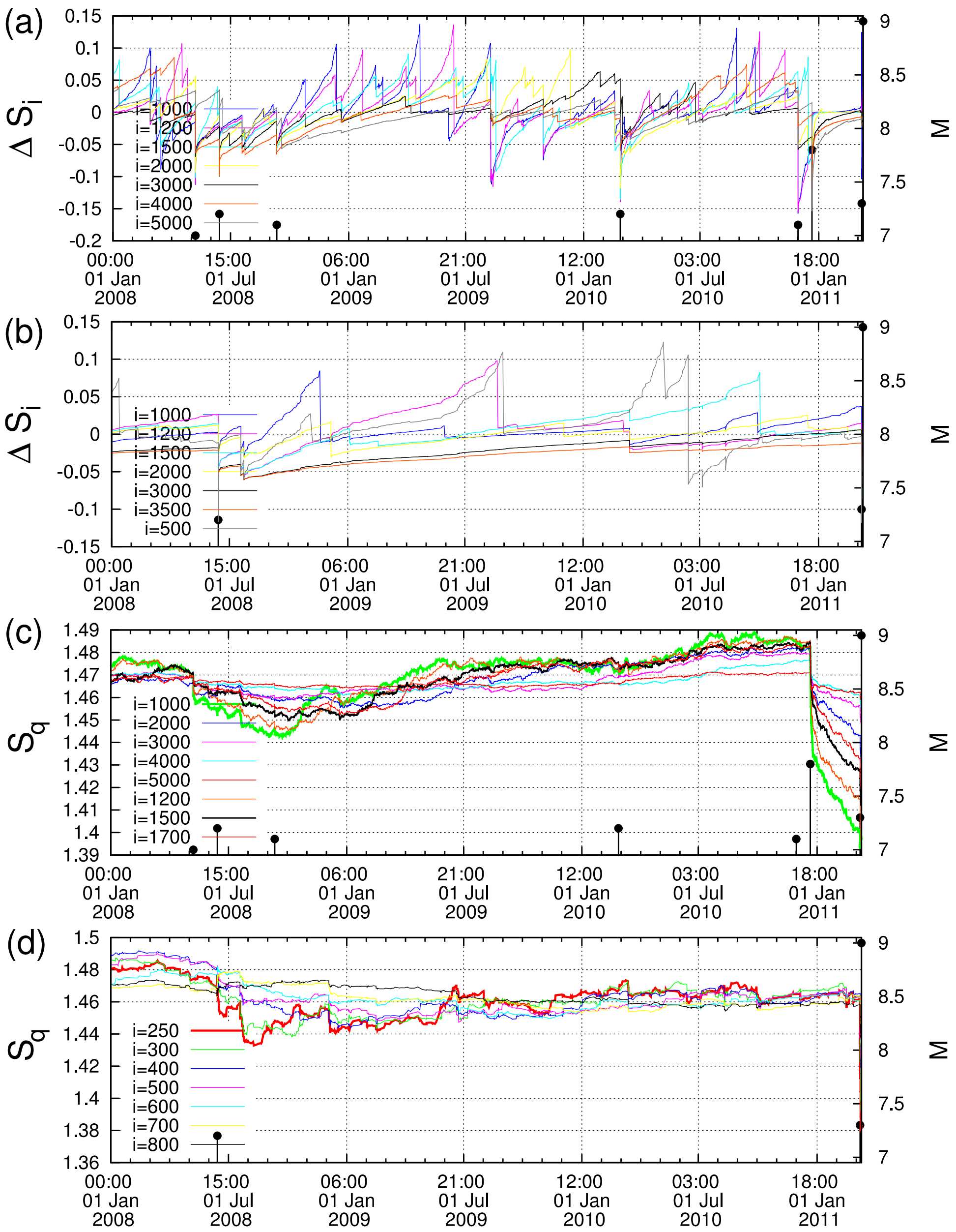

Fig. 3. Plot of the entropy change ΔS_i (a) for the entire Japanese area shown in Fig. 1(a), and (b) for the candidate epicentral area for the 2011 Tohoku $M9$ EQ depicted in Fig. 1(b) versus the conventional time. The same is plotted in (c) and (d) but for the Tsallis entropy S_q.

fluctuations of the entropy change under time-reversal for which the LSW prefactor A increases upon increasing the scale i [73].

Here, the following additional results emerge, see Figs. 3 and 4:

First, after the aforementioned period from 08:36 at 13:14 LT on 10 March 2011 in which the system entered the critical stage, we observe in Fig. 4(a)–(d) that simultaneous changes appear at 18:00 and 20:00 LT on 10 March 2011 on both the entropy change ΔS_i under time reversal in NTA and the Tsallis entropy S_q in NESM. Remarkably, these simultaneous changes are evident when computed in the future epicentral region but can be also seen – but with much difficulty – when the computation is made in the entire Japanese region. Second, a few hours later, the changes $\Delta \Lambda_i$ of all the complexity measures $\Delta \Lambda_{2000}$, $\Delta \Lambda_{3000}$ and $\Delta \Lambda_{4000}$ exhibit a simultaneous variation almost around 00:00 LT on 11 March

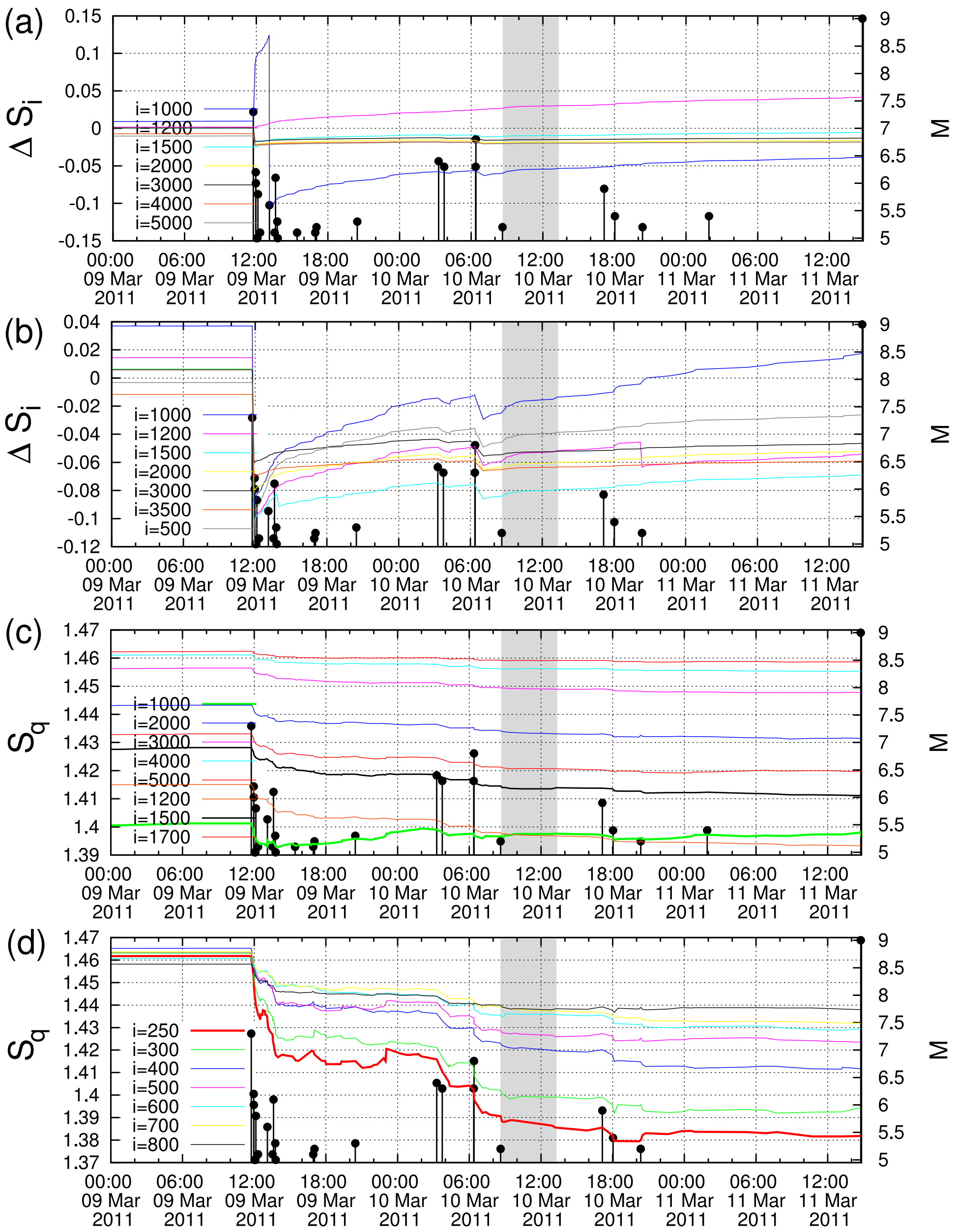

Fig. 4. The same as in Fig. 3 but in expanded time scale. The gray shaded area indicates the time period when true coincidence (i.e., the critical condition $\kappa_1 = 0.070$ holds) has been observed in Ref. [61], see also Fig. 2.

2011, i.e., several hours before the giant EQ occurrence, see Fig. 2. Third, Tsallis entropy S_q was found to exhibit scaling after the *M*7.3 foreshock occurrence on 9 March 2011 with a characteristic exponent 1/3 which conforms to the LSW phase transition theory as shown in Fig. 5.

5. Main conclusion

The combination of natural time analysis of seismicity with the non-extensive statistical mechanics enables the shortening of the time window of the impending 2011 Tohoku *M*9 EQ to several hours.

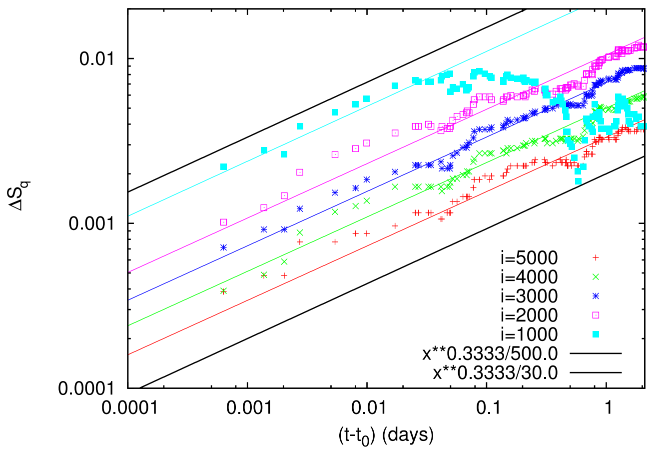

Fig. 5. Tsallis entropy shortly after (with $t_0 = 0.06$ day) the $M7.3$ foreshock occurrence on 9 March 2011 for various scales i.

CRediT authorship contribution statement

P.A. Varotsos: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Supervision, Project administration. **N.V. Sarlis:** Conceptualization, Methodology, Software, Formal analysis, Investigation, Resources, Writing –original draft, Writing – review & editing, Visualization. **E.S. Skordas:** Conceptualization, Methodology, Software, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Toshiyasu Nagao:** Conceptualization, Methodology, Validation, Investigation, Writing – review & editing. **Masashi Kamogawa:** Conceptualization, Methodology, Validation, Investigation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

References

[1] Huang Q. Seismicity changes prior to the Ms8.0 Wenchuan earthquake in Sichuan, China. Geophys Res Lett 2008;35:L23308. http://dx.doi.org/10.1029/2008GL036270.
[2] Huang Q. Retrospective investigation of geophysical data possibly associated with the Ms8.0 Wenchuan earthquake in Sichuan, China. J Asian Earth Sci 2011;41:421–7. http://dx.doi.org/10.1016/j.jseaes.2010.05.014.
[3] Telesca L, Lovallo M. Non-uniform scaling features in central Italy seismicity: A non-linear approach in investigating seismic patterns and detection of possible earthquake precursors. Geophys Res Lett 2009;36:001308. http://dx.doi.org/10.1029/2008GL036247.
[4] Lennartz S, Livina VN, Bunde A, Havlin S. Long-term memory in earthquakes and the distribution of interoccurrence times. EPL (Europhys Lett) 2008;81:69001. http://dx.doi.org/10.1209/0295-5075/81/69001.
[5] Lennartz S, Bunde A, Turcotte DL. Modelling seismic catalogues by cascade models: Do we need long-term magnitude correlations? Geophys J Int 2011;184:1214–22. http://dx.doi.org/10.1111/j.1365-246X.2010.04902.x.
[6] Rundle JB, Holliday JR, Graves WR, Turcotte DL, Tiampo KF, Klein W. Probabilities for large events in driven threshold systems. Phys Rev E 2012;86:021106. http://dx.doi.org/10.1103/PhysRevE.86.021106.
[7] Carlson JM, Langer JS, Shaw BE. Dynamics of earthquake faults. Rev Modern Phys 1994;66:657–70. http://dx.doi.org/10.1103/RevModPhys.66.657.
[8] Holliday JR, Rundle JB, Turcotte DL, Klein W, Tiampo KF, Donnellan A. Space–time clustering and correlations of major earthquakes. Phys Rev Lett 2006;97:238501. http://dx.doi.org/10.1103/PhysRevLett.97.238501.
[9] Varotsos PA, Sarlis NV, Skordas ES. Natural time analysis: the new view of time. Precursory seismic electric signals, earthquakes and other complex time-series, Berlin Heidelberg: Springer-Verlag; 2011, http://dx.doi.org/10.1007/978-3-642-16449-1.
[10] Turcotte DL. Fractals and chaos in geology and geophysics. 2nd ed.. Cambridge: Cambridge University Press; 1997, http://dx.doi.org/10.1017/CBO9781139174695.
[11] Varotsos PA, Sarlis NV, Skordas ES. Spatio-temporal complexity aspects on the interrelation between seismic electric signals and seismicity. Pract Athens Acad 2001;76:294–321.
[12] Varotsos PA, Sarlis NV, Skordas ES. Long-range correlations in the electric signals that precede rupture. Phys Rev E 2002;66:011902. http://dx.doi.org/10.1103/physreve.66.011902.
[13] Varotsos PA, Sarlis NV, Skordas ES. Seismic Electric Signals and Seismicity: On a tentative interrelation between their spectral content. Acta Geophys Pol 2002;50:337–54.

[14] Varotsos PA, Sarlis NV, Skordas ES, Uyeda S, Kamogawa M. Natural time analysis of critical phenomena. Proc Natl Acad Sci USA 2011;108:11361–4. http://dx.doi.org/10.1073/pnas.1108138108.
[15] Rundle JB, Turcotte DL, Donnellan A, Grant Ludwig L, Luginbuhl M, Gong G. Nowcasting earthquakes. Earth Space Sci 2016;3:480–6. http://dx.doi.org/10.1002/2016EA000185.
[16] Rundle JB, Luginbuhl M, Giguere A, Turcotte DL. Natural time, nowcasting and the physics of earthquakes: Estimation of seismic risk to global megacities. Pure Appl Geophys 2018;175:647–60. http://dx.doi.org/10.1007/s00024-017-1720-x.
[17] Luginbuhl M, Rundle JB, Hawkins A, Turcotte DL. Nowcasting earthquakes: A comparison of induced earthquakes in Oklahoma and at the Geysers, California. Pure Appl Geophys 2018;175:49–65. http://dx.doi.org/10.1007/s00024-017-1678-8.
[18] Luginbuhl M, Rundle JB, Turcotte DL. Natural Time and Nowcasting Earthquakes: Are Large Global Earthquakes Temporally Clustered? Pure Appl Geophys 2018;175:661–70. http://dx.doi.org/10.1007/s00024-018-1778-0.
[19] Tsallis C. Introduction to nonextensive statistical mechanics. Berlin: Springer; 2009, http://dx.doi.org/10.1007/978-0-387-85359-8.
[20] Tsallis C. Possible generalization of Boltzmann–Gibbs statistics. J Stat Phys 1988;52:479–87. http://dx.doi.org/10.1007/BF01016429.
[21] Tsallis C. The nonadditive entropy sq and its applications in physics and elsewhere: Some remarks. Entropy 2011;13(10):1765–804. http://dx.doi.org/10.3390/e13101765.
[22] Tsallis C. Enthusiasm and Skepticism: Two Pillars of Science–A Nonextensive Statistics Case. Physics 2022;4(2):609–32. http://dx.doi.org/10.3390/physics4020041.
[23] Penrose O. Foundations of statistical mechanics: a deductive treatment. Oxford, UK: Pergamon; 1970.
[24] Livadiotis G, McComas DJ. Thermodynamic Definitions of Temperature and Kappa and Introduction of the Entropy Defect. Entropy 2021;23(12):1683. http://dx.doi.org/10.3390/e23121683.
[25] Livadiotis G, McComas DJ. Physical Correlations Lead to Kappa Distributions. Astrophys J 2022;940(1):83. http://dx.doi.org/10.3847/1538-4357/ac99df.
[26] Livadiotis G. Kappa Distributions: Theory and Applications in Plasmas. Amsterdam Netherlands: Elsevier; 2017.
[27] Livadiotis G. Thermodynamic origin of kappa distributions. EPL (Europhys Lett) 2018;122(5):50001.
[28] Sotolongo-Costa O, Posadas A. Fragment–asperity interaction model for earthquakes. Phys Rev Lett 2004;92:048501. http://dx.doi.org/10.1103/PhysRevLett.92.048501.
[29] Silva R, França GS, Vilar CS, Alcaniz JS. Nonextensive models for earthquakes. Phys Rev E 2006;73:026102. http://dx.doi.org/10.1103/PhysRevE.73.026102.
[30] Vallianatos F, Karakostas V, Papadimitriou E. A non-extensive statistical physics view in the spatiotemporal properties of the 2003 (Mw6.2) Lefkada, Ionian Island Greece, aftershock sequence. Pure Appl Geophys 2014;171:1343–54. http://dx.doi.org/10.1007/s00024-013-0706-6.
[31] Michas G, Vallianatos F, Sammonds P. Statistical mechanics and scaling of fault populations with increasing strain in the Corinth Rift. Earth Planet Sci Lett 2015;431:150–63. http://dx.doi.org/10.1016/j.epsl.2015.09.014.
[32] Vallianatos F, Michas G, Papadakis G. A description of seismicity based on non-extensive statistical physics: A review. In: D'Amico Sebastiano, editor. Earthquakes and their impact on society. Cham: Springer International Publishing; 2016, p. 1–41. http://dx.doi.org/10.1007/978-3-319-21753-6_1.
[33] Saltas V, Vallianatos F, Triantis D, Stavrakas I. Complexity in laboratory seismology: From electrical and acoustic emissions to fracture. In: Chelidze T, Vallianatos F, Telesca L, editors. Complexity of seismic time series. Elsevier; 2018, p. 239–73. http://dx.doi.org/10.1016/B978-0-12-813138-1.00008-0.
[34] Vallianatos F, Michas G, Papadakis G. Nonextensive statistical seismology: An overview. In: Chelidze T, Vallianatos F, Telesca L, editors. Complexity of seismic time series. Elsevier; 2018, p. 25–59. http://dx.doi.org/10.1016/B978-0-12-813138-1.00002-X.
[35] Anyfadi E-A, Avgerinou S-E, Michas G, Vallianatos F. Universal Non-Extensive Statistical Physics Temporal Pattern of Major Subduction Zone Aftershock Sequences. Entropy 2022;24(12):1850. http://dx.doi.org/10.3390/e24121850.
[36] Lay T, Wallace TC. Modern Global Seismology. Elsevier; 1995.
[37] Darooneh AH, Mehri A. A nonextensive modification of the gutenberg-richter law: q-stretched exponential form. Physica A 2010;389:509.
[38] Telesca L, Lovallo M, Amin Mohamed AE-E, ElGabry M, El-hady S, Abou Elenean KM, ElShafey Fat ElBary R. Investigating the time-scaling behavior of the 2004–2010 seismicity of Aswan area (Egypt) by means of the Allan factor statistics and the detrended fluctuation analysis. Nat Hazards Earth Syst Sci 2012;12:1267–76. http://dx.doi.org/10.5194/nhess-12-1267-2012.
[39] Telesca L. Maximum Likelihood Estimation of the Nonextensive Parameters of the Earthquake Cumulative Magnitude Distribution. Bull Seismol Soc Am 2012;102:886–91. http://dx.doi.org/10.1785/0120110093.
[40] Kanamori H. Quantification of earthquakes. Nature 1978;271:411–4. http://dx.doi.org/10.1038/271411a0.
[41] Sarlis NV, Skordas ES, Varotsos PA. Nonextensivity and natural time: The case of seismicity. Phys Rev E 2010;82:021110. http://dx.doi.org/10.1103/physreve.82.021110.
[42] Gutenberg B, Richter CF. Frequency of earthquakes in California. Bull Seismol Soc Am 1944;34(4):185–8.
[43] Posadas A, Sotolongo-Costa O. Non-extensive entropy and fragment–asperity interaction model for earthquakes. Commun Nonlinear Sci Numer Simul 2023;117:106906. http://dx.doi.org/10.1016/j.cnsns.2022.106906.
[44] Brillinger DR. Time series, point processes, and hybrids. Can J Stat 1994;22(2):177–206. http://dx.doi.org/10.2307/3315583.
[45] Sarlis NV, Varotsos PA, Skordas ES. Flux avalanches in $YBa_2Cu_3O_{7-x}$ films and rice piles: Natural time domain analysis. Phys Rev B 2006;73:054504. http://dx.doi.org/10.1103/PhysRevB.73.054504.
[46] Loukidis A, Pasiou ED, Sarlis NV, Triantis D. Fracture analysis of typical construction materials in natural time. Physica A 2019;547:123831. http://dx.doi.org/10.1016/j.physa.2019.123831.
[47] Loukidis A, Perez-Oregon J, Pasiou ED, Kourkoulis SK, Sarlis NV, Triantis D. Natural time analysis of acoustic emissions before fracture: Results compatible with the Bak-Tang-Wiesenfeld model. Europhys Lett 2022;139(1):12004. http://dx.doi.org/10.1209/0295-5075/ac7bee.
[48] Varotsos PA, Sarlis NV, Tanaka HK, Skordas ES. Similarity of fluctuations in correlated systems: The case of seismicity. Phys Rev E 2005;72:041103. http://dx.doi.org/10.1103/physreve.72.041103.
[49] Sarlis NV, Skordas ES, Varotsos PA. Similarity of fluctuations in systems exhibiting Self-Organized Criticality. Europhys Lett 2011;96:28006. http://dx.doi.org/10.1209/0295-5075/96/28006.
[50] Loukidis A, Perez-Oregon J, Pasiou ED, Sarlis NV, Triantis D. Similarity of fluctuations in critical systems: Acoustic emissions observed before fracture. Physica A 2020;566:125622. http://dx.doi.org/10.1016/j.physa.2020.125622.
[51] Varotsos P, Alexopoulos K. Thermodynamics of point defects and their relation with bulk properties. Amsterdam: North Holland; 1986.
[52] Varotsos P. The physics of seismic electric signals. Tokyo: TERRAPUB; 2005.
[53] Varotsos P, Lazaridou M. Latest aspects of earthquake prediction in Greece based on Seismic Electric Signals. Tectonophysics 1991;188:321–47. http://dx.doi.org/10.1016/0040-1951(91)90462-2.
[54] Sarlis NV, Skordas ES, Lazaridou MS, Varotsos PA. Investigation of seismicity after the initiation of a Seismic Electric Signal activity until the main shock. Proc Jpn Acad Ser B Phys Biol Sci 2008;84:331–43. http://dx.doi.org/10.2183/pjab.84.331.
[55] Uyeda S, Kamogawa M. The Prediction of Two Large Earthquakes in Greece. Eos Trans AGU 2008;89:363. http://dx.doi.org/10.1029/2008EO390002.

[56] Varotsos PA, Sarlis NV, Skordas ES, Uyeda S, Kamogawa M. Natural time analysis of critical phenomena, the case of seismicity. Europhys Lett 2010;92:29002. http://dx.doi.org/10.1209/0295-5075/92/29002.
[57] Uyeda S, Kamogawa M, Tanaka H. Analysis of electrical activity and seismicity in the natural time domain for the volcanic-seismic swarm activity in 2000 in the Izu Island region, Japan. J Geophys Res 2009;114:B02310. http://dx.doi.org/10.1029/2007JB005332.
[58] Sarlis NV, Skordas ES, Varotsos PA, Nagao T, Kamogawa M, Tanaka H, Uyeda S. Minimum of the order parameter fluctuations of seismicity before major earthquakes in Japan. Proc Natl Acad Sci USA 2013;110:13734–8. http://dx.doi.org/10.1073/pnas.1312740110.
[59] Sarlis NV, Skordas ES, Varotsos PA, Nagao T, Kamogawa M, Uyeda S. Spatiotemporal variations of seismicity before major earthquakes in the Japanese area and their relation with the epicentral locations. Proc Natl Acad Sci USA 2015;112:986–9. http://dx.doi.org/10.1073/pnas.1422893112.
[60] Huang Q. Forecasting the epicenter of a future major earthquake. Proc Natl Acad Sci USA 2015;112:944–5. http://dx.doi.org/10.1073/pnas.1423684112.
[61] Varotsos PA, Sarlis NV, Skordas ES. Identifying the occurrence time of an impending major earthquake: a review. Earthq Sci 2017;30(4):209–18. http://dx.doi.org/10.1007/s11589-017-0182-7.
[62] Varotsos PA, Skordas ES, Sarlis NV. Fluctuations of the entropy change under time reversal: further investigations on identifying the occurrence time of an impending major earthquake. EPL (Europhys Lett) 2020;130:29001. http://dx.doi.org/10.1209/0295-5075/130/29001.
[63] Varotsos PA, Sarlis NV, Skordas ES, Lazaridou MS. Entropy in natural time domain. Phys Rev E 2004;70:011106. http://dx.doi.org/10.1103/physreve.70.011106.
[64] Varotsos PA, Sarlis NV, Skordas ES, Lazaridou MS. Natural entropy fluctuations discriminate similar-looking electric signals emitted from systems of different dynamics. Phys Rev E 2005;71:011110. http://dx.doi.org/10.1103/physreve.71.011110.
[65] Varotsos PA, Sarlis NV, Skordas ES, Lazaridou MS. Identifying sudden cardiac death risk and specifying its occurrence time by analyzing electrocardiograms in natural time. Appl Phys Lett 2007;91:064106. http://dx.doi.org/10.1063/1.2768928.
[66] Varotsos PA, Sarlis NV, Skordas ES. Attempt to distinguish electric signals of a dichotomous nature. Phys Rev E 2003;68:031106. http://dx.doi.org/10.1103/PhysRevE.68.031106.
[67] Varotsos PA, Sarlis NV, Tanaka HK, Skordas ES. Some properties of the entropy in the natural time. Phys Rev E 2005;71:032102. http://dx.doi.org/10.1103/physreve.71.032102.
[68] Sarlis NV, Christopoulos S-RG, Bemplidaki MM. Change ΔS of the entropy in natural time under time reversal: Complexity measures upon change of scale. Europhys Lett 2015;109:18002. http://dx.doi.org/10.1209/0295-5075/109/18002.
[69] Varotsos PA, Skordas ES, Sarlis NV. Fluctuations of the entropy change under time reversal: Further investigations on identifying the occurrence time of an impending major earthquake. EPL (Europhys Lett) 2020;130(2):29001. http://dx.doi.org/10.1209/0295-5075/130/29001.
[70] Sarlis NV, Skordas ES, Varotsos PA. The change of the entropy in natural time under time-reversal in the Olami-Feder-Christensen earthquake model. Tectonophysics 2011;513:49–53. http://dx.doi.org/10.1016/j.tecto.2011.09.025.
[71] Perez-Oregon J, Angulo-Brown F, Sarlis NV. Nowcasting Avalanches as Earthquakes and the Predictability of Strong Avalanches in the Olami-Feder-Christensen Model. Entropy 2020;22(11):1228. http://dx.doi.org/10.3390/e22111228.
[72] Sarlis NV, Skordas ES, Varotsos PA. A remarkable change of the entropy of seismicity in natural time under time reversal before the super-giant M9 Tohoku earthquake on 11 March 2011. EPL (Europhys Lett) 2011;124(2018):29001. http://dx.doi.org/10.1209/0295-5075/124/29001.
[73] Varotsos PA, Sarlis NV, Skordas ES. Tsallis Entropy Index q and the Complexity Measure of Seismicity in Natural Time under Time Reversal before the M9 Tohoku Earthquake in 2011. Entropy 2018;20(10):757. http://dx.doi.org/10.3390/e20100757.
[74] Lifshitz I, Slyozov V. The kinetics of precipitation from supersaturated solid solutions. J Phys Chem Solids 1961;19(1):35–50. http://dx.doi.org/10.1016/0022-3697(61)90054-3.
[75] Wagner C. Theorie der alterung von niederschlägen durch umlösen (ostwald-reifung). Z Elektrochem Berichte Bunsenges Phys Chem 1961;65:581–91. http://dx.doi.org/10.1002/bbpc.19610650704.
[76] Varotsos PA, Sarlis NV, Skordas ES, Nagao T, Kamogawa M. The unusual case of the ultra-deep 2015 Ogasawara earthquake (M_W7.9): Natural time analysis. Europhys Lett 2015;135(2021):49002. http://dx.doi.org/10.1209/0295-5075/135/49002.
[77] Tanaka HK, Varotsos PA, Sarlis NV, Skordas ES. A plausible universal behaviour of earthquakes in the natural time-domain. Proc Jpn Acad Ser B Phys Biol Sci 2004;80:283–9. http://dx.doi.org/10.2183/pjab.80.283.
[78] Varotsos PA, Sarlis NV, Skordas ES, Lazaridou MS. Seismic Electric Signals: An additional fact showing their physical interconnection with seismicity. Tectonophysics 2013;589:116–25. http://dx.doi.org/10.1016/j.tecto.2012.12.020.
[79] Varotsos P, Alexopoulos K, Lazaridou M. Latest aspects of earthquake prediction in Greece based on Seismic Electric Signals, II. Tectonophysics 1993;224:1–37. http://dx.doi.org/10.1016/0040-1951(93)90055-O.
[80] Varotsos PA, Sarlis NV, Skordas ES, Lazaridou MS. Fluctuations, under time reversal, of the natural time and the entropy distinguish similar looking electric signals of different dynamics. J Appl Phys 2008;103:014906. http://dx.doi.org/10.1063/1.2827363.
[81] Varotsos PA, Sarlis NV, Skordas ES. Long-range correlations in the electric signals the precede rupture: Further investigations. Phys Rev E 2003;67:021109. http://dx.doi.org/10.1103/PhysRevE.67.021109.
[82] Varotsos PA, Sarlis NV, Skordas ES. Study of the temporal correlations in the magnitude time series before major earthquakes in Japan. J Geophys Res: Space Phys 2014;119:9192–206. http://dx.doi.org/10.1002/2014JA020580.
[83] Peng C-K, Buldyrev SV, Havlin S, Simons M, Stanley HE, Goldberger AL. Mosaic organization of DNA nucleotides. Phys Rev E 1994;49:1685–9. http://dx.doi.org/10.1103/physreve.49.1685.
[84] Taqqu MS, Teverovsky V, Willinger W. Estimators for long-range dependence: An empirical study. Fractals 1995;3:785–98.
[85] Skordas ES, Sarlis NV, Varotsos PA. Precursory variations of Tsallis non-extensive statistical mechanics entropic index associated with the M9 Tohoku earthquake in 2011. Eur Phys J Spec Top 2020;229:851–9. http://dx.doi.org/10.1140/epjst/e2020-900218-x.