Supplementary information on: Time-difference between the electric field components of signals prior to major earthquakes

P. A. Varotsos,1,2 N. V. Sarlis,1 and E. S. Skordas1,2
1Solid State Section and Solid Earth Physics Institute, Physics Department, University of Athens, Panepistimiopolis, Zografos 157 84, Athens, Greece

The sites of the electrodes of the dipoles analyzed in the main text are presented in Section I. Section II describes in short the filters used for the electric field measurements. In Section III, we give details on the procedure for the calculation of the electromagnetic fields, in the time-domain, for the case of an electric dipole source surrounded by a homogeneous conductive medium. The corresponding details for a dipole source lying inside (and parallel to the main axis of) a conductive cylinder embedded in a more resistive medium are given in Section IV. Finally, in section V we offer some comments on the point that nowadays it is experimentally accessible to determine characteristics of short duration pulses by using repetitive sample rate with a period wider than the pulse.

PACS numbers: 91.25.Qs, 91.30.Fx

I. THE SITES OF THE ELECTRODES

The electric field measurements have been carried out by several dipoles with lengths a few to several tens of meters (short dipoles) or a couple of kilometers (long dipoles). The short- and long- dipoles are measured by “10Hz” and “3Hz” low pass filters, respectively. A map of all these dipoles is given in Fig.1 of the supplementary information of Ref.1. Here, Fig.1 depicts the sites of the electrodes of the short dipoles’ array. Only the data of the dipoles $E_c - W_c$ and $N_c - S_c$ (i.e., the two dipoles to the left of the figure) are analyzed here for reasons described in the main text. The magnetometers are located close to the middle of the dipole labelled $N'S'$, which lies between the sites ‘b’ and ‘c’.

II. THE FILTERS FOR THE ELECTRIC FIELD MEASUREMENTS

The so-called low pass “10Hz” filters used, are fourth order active low pass filters, having two symmetric second order poles in the complex $(i^2 = -1)$ $f$-plane with a frequency response:

$$ R(f) = \frac{A \exp(-2\pi i f \tau_d)}{[1 - (f/f_p)^2 + i \sqrt{2(f/f_p)}]^2} \quad (1) $$

where $A$ is the amplification, $f_p$ is the 3dB frequency corner of the filter, and $\tau_d$ is the time delay of the filter. The expression (1) was applied to the laboratory measured data for both amplitude and phase, and a non-linear least squares fitting was performed (see Fig.2) using the constant chi-square ($\nu = 95\%$) boundaries for the determination of the errors in the fitting parameters (see Table 1).

Finally, Eq. (1) leads to an impulse response:

$$ I(t) = \frac{2A}{T_p} \exp \left[ \frac{\left(\frac{t - \tau_d}{T_p}\right)}{T_p} \right] \left[ \sin \left( \frac{t - \tau_d}{T_p} \right) \right] \cos \left( \frac{t - \tau_d}{T_p} \right) \Theta(t - \tau_d) \quad (2) $$

where $T_p = 14.6 \pm 0.4\text{ms}$ and $\tau_d = 1.5 \pm 0.4\text{ms}$, determined by the aforementioned laboratory calibration see Fig.2.

<table>
<thead>
<tr>
<th>Type of filter</th>
<th>$A$ (Hz)</th>
<th>$f_p$ (Hz)</th>
<th>$\tau_d$ (ms)</th>
<th>$T_p$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“10Hz”</td>
<td>1.015±0.024</td>
<td>15.4±0.4</td>
<td>1.5±0.4</td>
<td>14.6±0.4</td>
</tr>
</tbody>
</table>

TABLE 1: The amplification $A$, the 3dB frequency corner $f_p$, the time delay $\tau_d$, and the characteristic “period” $T_p$ for the low-pass filters used in the field experiments.
FIG. 1: Configuration of short dipoles at IOA. The short dipoles with subscripts c are those located at the site ‘c’ which have been analyzed in the main text. Concerning the geology: (1) alluvial deposits, (2) flysch of the Ioanian unit, (3) limestones. The geographical coordinates of the station are $39^\circ42'\,N,20^\circ51'.E$ (Northwestern Greece).

III. THE CASE OF A HOMOGENEOUS CONDUCTIVE MEDIUM

The electric field components, in cylindrical coordinates, for a current density $j(r, t) = f(t)^3(r)\hat{z}$, localized at the origin, are given by: \(^2\)

$$E_z(r, t) = \frac{\mu}{4\pi^2 r^2 \tau^4} [I_{5/2} - \sin^2(\theta)I_{7/2}],$$

$$E_r(r, t) = \frac{\mu}{4\pi^2 r^2 \tau^4} \sin(\theta) \cos(\theta)I_{7/2},$$

where $\mu$ is the magnetic permeability, $(r, \theta, \phi)$ are the spherical coordinates of $r$, $\tau_0 = \mu\sigma r^2/4$ and:

$$I_{\nu} = \int_{-\infty}^{t} \left[\frac{\tau_0}{(t-t')}\right]^\nu \exp\left[-\frac{\tau_0}{(t-t')}\right] f(t') dt'.$$

Equations (3) and (4), imply that the time-evolution of the electric field components could be solely described in terms of $I_{5/2}$ and $I_{7/2}$, if the electric field components are measured (in two perpendicular directions) at a site located at small $\theta$. Assuming $f(t) = \Theta(t)$, where $\Theta(t)$ is the Heaviside unit-step function, we can directly integrate Eq.(5) to obtain:

$$H_{5/2}(t) = \frac{I_{5/2}}{\tau_0} = x \exp(-x^2) + \frac{\sqrt{\pi}}{2} \text{erfc}(x),$$

$$H_{7/2}(t) = \frac{I_{7/2}}{\tau_0} = \left(x^2 + \frac{3}{2}x^2\right) \exp(-x^2) + \frac{3\sqrt{\pi}}{4} \text{erfc}(x),$$

P. A. Varotsos, N. V. Sarlis and E. S. Skordas

March 16, 2005
where \( x = \sqrt{\tau_0/\ell} \), and erf(-w) = \( \frac{2}{\sqrt{\pi}} \int_0^\infty \exp(-z^2)dz \) the complementary error function. These equations reveal that the electric field components have different time evolutions, which are described by \( H_{5/2} \) and \( H_{7/2} \). Their feature for realistic \( \tau_\ell \) values, can be visualized in Fig.3, where we also plot the functions \( F_{5/2} \) and \( F_{7/2} \), which correspond to \( H_{5/2} \) and \( H_{7/2} \), respectively, and show the recordings after the “10Hz” low pass filter (i.e., after their convolution with the filter’s impulse response, described by Eq. (2)). They exhibit an overall behavior, which is, more or less, consistent with the feature of our experimental results, i.e., when comparing the \( \tau_\ell \) values estimated from Fig.3 for \( F_{5/2} \) and \( F_{7/2} \) with the experimental \( \tau_\ell \) values -for \( E_c-W_c \) and \( N_c-S_c \), respectively- we find that they are, more or less, comparable.

IV. A CONDUCTIVE CYLINDER EMBEDDED IN A MEDIUM OF SMALLER CONDUCTIVITY

We now study a current dipole source lying inside and parallel to the main axis \( z \) of a conductive cylinder (radius \( R \), conductivity \( \sigma_p \)) of infinite length (the cylinder is assumed to be surrounded by a less conductive medium of conductivity \( \sigma \)). This calculation reveals that at long (reduced) distances from the emitting source, and for sites lying inside the more resistive medium but very close to the cylinder, the component \( E_\ell \), normal to the cylinder axis has a \( \tau_\ell \) value appreciably smaller than that of the component \( E_z \) (along the cylinder axis), see Fig.4 of Ref.1. Their difference,
FIG. 3: The calculated values of the time evolution (normalized to unity at infinite time) of the “fast”, $H_{7/2}$, and “slow”, $H_{5/2}$, electric field variations, for the case $\tau \approx 0.16$ sec (corresponding to $\mu = \mu_0, \sigma = 500 \text{m}$, and $\rho = 1/\sigma = 4000 \text{m}^{-1}$). The thick lines $H_{7/2}$ and $H_{5/2}$ depict the values measured for $H_{7/2}$ and $H_{5/2}$, respectively, when using a “10Hz” low pass filter. The limits of 15% and 85% are also drawn.

for reasonable conductivity ratios, e.g., $\sigma_p/\sigma \approx 4000/10$, and for distances of the order of 100 km ($R \approx 500 \text{m}$), is of the order of some tenths of a sec, and hence measurable. (See also Fig. 4 of Ref. 1; no frequency dispersion of the dielectric constant was assumed in all the calculations throughout this letter as in Ref. 1). This is reminiscent of our experimental results.

The most probable model for the SES, however, requires the calculation of the electric field close to the edge of a highly conductive path, but when the current dipole source is oriented almost perpendicular to the neighboring conductive path$^{1,3}$ (Note that the existence of such conductive paths in the neighborhood of the emitting dipole electric current source - which is produced in the focal area of the impending seismic event due to the increasing stress - is highly probable in view of the following fact: Recent detailed independent measurements$^{2}$ reveal that strike-slip fault zones are actually often associated with electrically conductive structures in the brittle crust). Such a calculation, however, is not currently available. In view of the lack of such a calculation for the latter heterogeneous structure, we restrict ourselves in the main text to the comparison of the experimental data with the theoretical behavior in a homogeneous conductive medium, depicted in Fig. 3 of the main text.

V. COMMENTS ON OBTAINING CHARACTERISTICS OF SHORT DURATION PULSES BY USING REPETITIVE SAMPLING WITH A PERIOD WIDER THAN THE PULSE

Here we offer some clarifications on the plausibility of determining characteristic times $\tau_p$ of 140 msec or 320 msec from the analysis of several ($N=500$) pulses. We clarify that during the last decade well known techniques (e.g., modern Oscilloscope techniques) have been developed that allow the accurate measurement of the characteristics of a short duration pulse by using repetitive sample rate that has a period wider than the pulse (e.g., see Fig. 4-28 of NATIONAL INSTRUMENTS Developer Zone entitled Repetitive Sampling by R.A. Witte (available at: http://zone.ni.com/devzone/conceptd.nsf/webmain/FOF361DA2C55870806862568430078425), where an example is given how “a 30 msec is accurately measured even though the random repetitive sample rate of 10 Msamples/sec has a period wider than the pulse”; see also Tektronix Sampling Oscilloscope Techniques in TECHNIQUE PRIMER 47W-7200, available at: http://ww.tek.com/Measurement/cgi-bin/framed.pl?Document=Measurement/App_Notes/sampling_primer&type=1&FrameSet=oscilloscopes). In other words, the nowadays knowledge, fully justifies the procedure we presented in the main text, which basically consists of the following: Upon having recorded a large amount of randomly triggered pulses (500) with a sampling rate 1 sample/sec, we can determine characteristic times ($\tau_p$) of 140
msc or 320 msc.

---

* Electronic address: pvare@otenet.gr