Appendix 4.

The physical meaning of ΔS_i

We present a simple example in which the meaning of the entropy change ΔS under time reversal seems to emerge clearly.

Let us consider the parametric family: $p(\chi;\varepsilon) = 1+\varepsilon(\chi - 1/2)$ for small ε (<1), where $p(\chi)$ is a continuous probability density function (PDF) corresponding to the point probabilities p_k used so far. Such a family of PDFs shares the interesting property $\mathcal{T}p(\chi;\varepsilon) = p(\chi;-\varepsilon)$, i.e, the action of time reversal is obtained by simply changing the sign of ε. Moreover, the calculation of the entropy $S(\varepsilon) \equiv S[p(\chi;\varepsilon)] = \langle \chi \ln \chi \rangle - \langle \chi \rangle \ln \langle \chi \rangle$, where $\langle f(\chi) \rangle = \int_0^1 p(\chi;\varepsilon) f(\chi) d\chi$, can be done analytically, and the result yields[1]

$$S(\varepsilon) = -\frac{1}{4} + \frac{\varepsilon}{72} - \left(\frac{1}{2} + \frac{\varepsilon}{12}\right) \ln\left(\frac{1}{2} + \frac{\varepsilon}{12}\right). \qquad (1)$$

An interrelation between $\Delta S(\varepsilon) = S(\varepsilon) - S(-\varepsilon)$ and the small linear trend parameter ε, can be obtained by expanding Eq.(1) around ε=0, which leads to

$$\Delta S(\varepsilon) = \left(\frac{6\ln 2 - 5}{36}\right)\varepsilon + O(\varepsilon^3) \qquad (2)$$

Since ln2 < 5/6, Eq.(2) implies that a positive ε (i.e., increasing trend) corresponds to negative ΔS and vice versa. Thus, the variation of the ΔS_i in the ECG data may be thought as capturing the net result, at scale i, of the competing mechanisms that decrease or increase heart rate. Concerning these mechanisms, recall the following point discussed in the last but one paragraph of the main text:

Physiologically, the origin of the complex dynamics of heart rate has been attributed to *antagonistic* activity of the two branches of the autonomic nervous system, i.e., the parasympathetic and the sympathetic nervous systems, respectively, decreasing and increasing heart rate[2,3,4]. Their net result seems to be actually captured by ΔS_i in accordance to Eq.(2),

If we consider[5] that S could be thought of as a measure of the "disorder" (in successive intervals) and that the essence of the natural time-domain analysis is built on the variation of the durations of consecutive pulses, we may say the following: when approaching sudden death, the difference between the "disorder" looking in the (immediate) future, i.e., S, and that in the (immediate) past, i.e., S_-, becomes in SD of profound importance when compared to the corresponding difference under truly healthy conditions.

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