Supplementary information for the Brief Report 'Multiplicative cascades and seismicity in natural time'

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Details on the derivation of Eq.(1) of the main text are provided along with a schematic representation in natural time of the multiplicative cascade model for earthquakes. Furthermore, details on the temporal correlations obtained from real earthquake data sets are given.

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A schematic representation in natural time of the multiplicative cascade model for earthquakes, discussed in the main text, can be seen in Fig.1 Details on the proof of Eq.(1) of the main text are given in Section I. The tentative determination[1] of the b-value from first principles is shortly commented on in Section II. The presence of temporal correlation, detected by means of natural time as well as by the method presented in Ref.[2], upon changing the magnitude threshold of the catalogue is described in Section III.

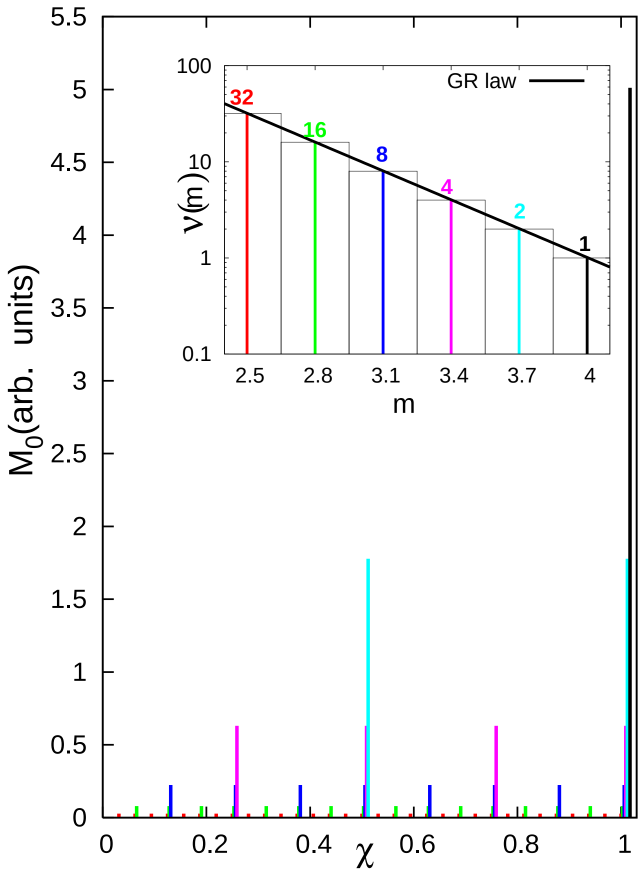

FIG. 1: (color online) A series of earthquakes that obey the GR law (see the inset) with magnitudes varying in the range $[2.5, 4]$ arranged in natural time according to their frequency $\nu(m)$. They form a multiplicative cascade, a dCs in this case. The corresponding values of the seismic moment, which is proportional to the energy release, are according to Ref.[3].

I. THE NORMALIZED POWER SPECTRUM IN NATURAL TIME FOR THE DETERMINISTIC CANTOR SET (DCS)

In the case of dCs with equal segments, the analysis in natural time simplifies, actually $\Pi(\omega)$ factorizes[1], which gives the advantage of an analytic calculation of the value of κ_1. For example, let us consider the initial natural time interval $\chi \in [0, 1]$ and assume the weights $w_i, i = 1, 2, ..., K$ equal to the probabilities p_k. Then, we have at the stage $M = 1$:

$$\Phi_1(\omega) = \sum_{j=1}^{K} w_j \exp\left(i\omega \frac{j}{K}\right), \tag{1}$$

whereas at stage $M = 2$:

$$\Phi_2(\omega) = \sum_{j=1}^{K} \sum_{k=1}^{K} w_k w_j \exp\left[i\omega\left(\frac{j-1}{K} + \frac{k}{K^2}\right)\right]. \tag{2}$$

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Rearranging the exponents in Eq.(2), we obtain:

$$\Phi_2(\omega) = \sum_{j=1}^{K}\sum_{k=1}^{K} w_k w_j \exp\left[i\left(\frac{\omega}{K}\frac{k}{K}+\omega\frac{j-1}{K}\right)\right]$$
$$= \left[\sum_{k=1}^{K} w_k \exp\left(i\frac{\omega}{K}\frac{k}{K}\right)\right]\left[\sum_{j=1}^{K} w_j \exp\left(i\omega\frac{j-1}{K}\right)\right]$$
$$= \Phi_1\left(\frac{\omega}{K}\right)\left[\sum_{j=1}^{K} w_j \exp\left(i\omega\frac{j-1}{K}\right)\right]. \quad (3)$$

Generalizing to stage M, we get the recursive formula:

$$\Phi_M(\omega) = \Phi_{M-1}\left(\frac{\omega}{K}\right)\left[\sum_{j=1}^{K} w_j \exp\left(i\omega\frac{j-1}{K}\right)\right]. \quad (4)$$

Equation (4) simply reflects the iterative procedure from the stage $M-1$ to the stage M. This can be considered as a two actions process: first, shrinking the representation of the interval of $\chi \in [0,1]$ at stage $M-1$ from [0,1] to $[0,1/K]$ by using $\Phi_{M-1}(\omega/K)$ and second expanding back to $\chi \in [0,1]$ by multiplying with the appropriate weighted factor, i.e., the term in the square brackets in Eq.(4). By virtue of Eq.(4), the normalized power spectrum $\Pi_M(\omega) \equiv |\Phi_M(\omega)|^2$ factorizes in the sense:

$$\Pi_M(\omega) = \Pi_{M-1}\left(\frac{\omega}{K}\right)\left|\sum_{j=1}^{K} w_j \exp\left(i\omega\frac{j-1}{K}\right)\right|^2. \quad (5)$$

Equation (5) is Eq.(1) of the main text.

II. DETERMINATION OF THE $b-$VALUE FROM FIRST PRINCIPLES

Equation (3) of the main text should be considered in conjuction with the following interconnection between κ_1 and b obtained in Ref.[1]: upon just assuming that a system emits bursts of various energies obeying a power-law distribution, the differential entropy associated with the probability $P(\kappa_1)$ versus κ_1 maximizes for $b \approx 1$, which is the value determined from real earthquake data.

III. ON THE PRESENCE OF TEMPORAL CORRELATION UPON CHANGING THE MAGNITUDE THRESHOLD OF THE CATALOGUE

The presence of temporal corelations is here investigated upon changing the magnitude threshold in the SCEC catalogue for the period from 1981-2003. This catalogue refers to the area $\mathrm{N}_{32}^{37}\mathrm{W}_{114}^{122}$, and according to Ref.[4] is complete above $M_{thres} = 1.8$ since 1981; this is why we select the period 1981-2003 in our investigation.

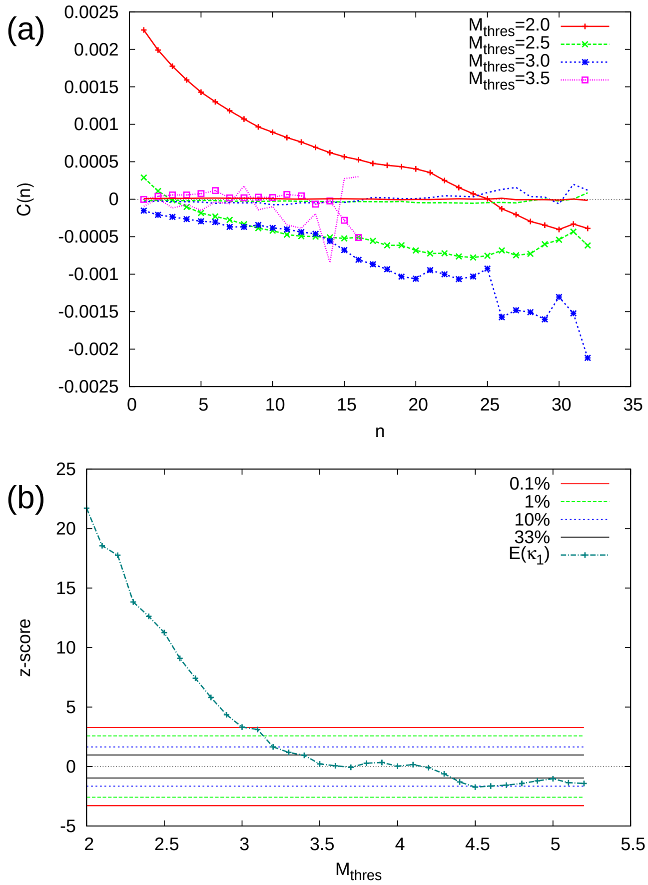

FIG. 2: (color online) (a) The values of $C(n)$ for various magnitude thresholds M_{thres} for the original data of SCEC(lines with points) together with those obtained(lines without points) when randomly shuffling the same data. For $M_{thres} = 3.5$, the quantity n_{max} was lowered to 16 due to the relatively small size of the corresponding catalogue. (b) The z-score of $\mathrm{E}(\kappa_1)$ of the original SCEC data calculated with respect to the Gaussian distribution $N(\mu,\sigma)$ of $\mathrm{E}(\kappa_{1,shuf})$ (see the text) as a function of M_{thres}. The intervals corresponding to the probability $\mathcal{P}$ to observe the correlation present in the original sequence of events by chance are bounded by the horizontal lines for $\mathcal{P} = 10^{-3}, 10^{-2}, 10^{-1}$ and 0.33.

For each M_{thres}, the catalogue was randomly shuffled and the distribution of $\mathrm{E}(\kappa_{1,shuf})$ was determined. It turned out that for every M_{thres} considered, $\mathrm{E}(\kappa_{1,shuf})$ exhibits a Gaussian distribution $N(\mu,\sigma)$ with average value μ and standard deviation σ. Both parameters μ and σ depend on M_{thres}.

In order to quantify the extent to which the original data exhibit temporal correlations versus the magnitude threshold, we plot in Fig.2(b) the z-score ($z \equiv (\mathrm{E}(\kappa_1)-\mu)/\sigma$) of $\mathrm{E}(\kappa_1)$ for the original catalogue with respect to the Gaussian distribution of $\mathrm{E}(\kappa_{1,shuf})$. If the z-score differs markedly from zero, this indicates the presence of temporal correlations. Figure 2(b) reveals the following: A clear descending initial part in the magnitude threshold range M_{thres} =2 to 3.1, which indicates a gradual decrease in the statistically *significant* temporal correlations. This result is consistent with the expectation that upon increasing the magnitude threshold, the number of aftershocks involved in the calculation decreases. For the sake of comparison, we depict in Fig.2(a), the values of $C(n)$ which interestingly exhibit the same trend. For larger values of M_{thres} *no definite* results can be statistically inferred for the presence of temporal correlations in the catalogues.

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[4] See the document `/SCSN/README.old` included in `SCSN_catalogs.tar.gz` available at `http://www.data.scec.org/ftp/catalogs/SCSN/`.