Multiplicative cascades and seismicity in natural time

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Natural time χ enables the distinction of two origins of self-similarity, i.e., the process memory and the process increments *infinite* variance. Employing multiplicative cascades in natural time, the most probable value of the variance $\kappa_1(\equiv\langle\chi^2\rangle-\langle\chi\rangle^2)$ is explicitly related with the parameter b of the Gutenberg-Richter law of randomly shuffled earthquake data. Moreover, the existence of temporal and magnitude correlations is studied in the original earthquake data. Magnitude correlations are larger for closer in time earthquakes, when the maximum interoccurrence time varies from half a day to 1 min.

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A large variety of natural systems exhibit irregular and complex behavior which in fact possesses scale-invariant structure [1,2]. In several systems, this nontrivial structure points to long-range temporal correlations which alternatively means that self-similarity results from process' memory only (but we stress that long-range temporal correlations do not automatically imply self-similarity of a process, e.g., [3], see also below). This is the case, for example, of fractional Brownian motion or of seismic electric signal (SES) activities. The latter are transient low frequency (≤ 1 Hz) electric signals emitted before earthquakes [4,5] presumably arising from a cooperative orientation of electric dipoles formed due to defects [6] when the stress in the focal area reaches a *critical* value [7]. Alternatively, the self-similarity may solely result from the process' increments infinite variance. This is the case, for example, of Levy stable motion. Note that Levy stable distributions, which are followed by many natural processes (e.g., see [8]), have heavy tails and their variance is infinite [9–11]. In general, the distinction of these two origins of self-similarity, i.e., process' memory and process' increments "infinite" variance—which may coexist—is a difficult task. This has been attempted in Ref. [3] by employing the natural time analysis and is further investigated here.

In a time series consisting of N events, the *natural time* $\chi_k=k/N$ serves as an index [12,13] for the occurrence of the kth event. The evolution of the pair (χ_k,Q_k) is studied, where Q_k denotes a quantity proportional to the energy released in the kth event. In the analysis of seismicity [13,14], Q_k may be considered as the seismic moment M_{0k} of the kth event, since M_0 is roughly proportional to the energy released during an earthquake (EQ). It has been shown [15] that in time series analysis, natural time reduces uncertainty and extracts signal information as much as possible. The normalized power spectrum in natural time is defined as $\Pi(\omega)\equiv|\Phi(\omega)|^2$, where $\Phi(\omega)=\sum_{k=1}^{N}p_k\exp(i\omega k/N)$. In this definition, p_k stands for $p_k=Q_k/\sum_{n=1}^{N}Q_n$ and $\omega=2\pi\phi$; where ϕ denotes the *natural frequency*. For $\omega\to 0$, $\Pi(\omega)$ simplifies to $\Pi(\omega)=1-\kappa_1\omega^2+\cdots$ where $\kappa_1=\sum_{k=1}^{N}p_k\chi_k^2-(\sum_{k=1}^{N}p_k\chi_k)^2$, i.e., the variance of natural time χ: $\kappa_1=\langle\chi^2\rangle-\langle\chi\rangle^2$, where $\langle f(\chi)\rangle=\sum_{k=1}^{N}p_kf(\chi_k)$. As shown in Ref. [14], $\Pi(\omega)$ for $\omega\to 0$ (or κ_1) can be considered as an order parameter for seismicity since its value changes abruptly when a main shock occurs and the statistical properties of its fluctuations resemble those in other nonequilibrium systems [16,17] (e.g., three-dimensional turbulent flow) as well as in equilibrium critical phenomena (e.g., two-dimensional Ising model [18]). For the SES activities that exhibit *infinitely* ranged temporal correlations, the following relation holds [12]: $\kappa_1=0.070$. Apart from $\Pi(\omega)$ or κ_1, another useful quantity in natural time is the entropy S, which is defined as $S\equiv\langle\chi\ln\chi\rangle-\langle\chi\rangle\ln\langle\chi\rangle$. This quantity depends on the sequential order of events [19,20] and exhibits [21] concavity, positivity, and Lesche [22] stability. The S value becomes equal to $\ln 2/2-1/4\approx 0.0966$ for a "uniform" (u) distribution, e.g., when all p_k are equal or Q_k are positive independent and identically distributed random variables of finite variance [19]. In this case, κ_1 and S are designated $\kappa_u(=1/12)$ and $S_u(=\ln 2/2-1/4)$, respectively. The same holds for the value of the entropy obtained [21,23] upon considering the time reversal $\mathcal{T}$, i.e., $\mathcal{T}p_k=p_{N-k+1}$, which is labeled by S_-.

The use of natural time analysis can lead to the identification of the origin of self-similarity as follows [3]: first, if self-similarity results from the process' memory *only*, the κ_1 value should change to $\kappa_u=1/12$ (and the values of S, S_- to $S_u=0.0966$) for the (randomly) shuffled data. Second, if the self-similarity *exclusively* results from process' increments infinite variance, the $\kappa_{1,p}$ value, at which the probability distribution $P(\kappa_1)$ maximizes, should be the same (but different from κ_u) for the original and the randomly shuffled data. This procedure answers, for example, to the fundamental problem of distinguishing between stochastic models characterized by different statistics, e.g., between fractal Gaussian intermittent noise and Levy-walk intermittent noise, which may equally well reproduce some patterns of a time series [10,24]. When *both* sources of self-similarity are present in the time series, as in the case of the time series analyzed here, quantitative conclusions on their relative strength can be obtained on the basis of Eqs. (12) and (13) of Ref. [3] as will be shown in the present Brief Report.

The aforementioned procedure of natural time was used in Ref. [3] to investigate the case of real earthquake data, for which several studies point to the conclusion that exhibit

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complex patterns of magnitude, spatial, and temporal correlations [25–32]. By calculating the κ_1 value in a window $l=6$ to 40 consecutive events sliding through either the original earthquake catalog or a shuffled one the following results have been obtained, for the Southern California Earthquake Catalog (SCEC) as well as for the Japanese Meteorological Agency Earthquake Catalog (Japan). Comparing the $\kappa_{1,p}$ values, we find that $\kappa_{1,p} \approx 0.066$ for the original data while $\kappa_{1,p} \approx 0.064$ for the surrogate data. Both these $\kappa_{1,p}$ values (with a plausible uncertainty of ± 0.001) differ markedly from the value $\kappa_u = 1/12$ of the uniform distribution. This was interpreted as reflecting that the self-similarity mainly originates from the process' increments infinite variance. Additionally, since the $\kappa_{1,p}$ value of the original earthquake data does not differ greatly from the value $\kappa_1 \approx 0.070$ found [3] in infinitely ranged temporal correlations, this indicates the importance of temporal correlations rather than their absence in the earthquake catalogs. In other words, the temporal correlations are responsible for the difference between the value of $\kappa_{1,p} \approx 0.064$ of the surrogate data from the value of $\kappa_{1,p} \approx 0.066$ of the original data.

The aim of the present Brief Report is twofold: first, to demonstrate theoretically that for (randomly) shuffled earthquake data that obey the Gutenberg-Richter (GR) law [i.e., the (cumulative) number of EQs with magnitude greater than or equal to m_0 occurring in a specified area and time is given by $N(m \geq m_0) \propto 10^{-bm_0}$, where b is a constant (≈ 1, e.g., [33])], there is an *explicit* interrelation between the most probable value of κ_1, i.e., $\kappa_{1,p}$, and the parameter b. Second, to further shed light on the presence of temporal correlations in seismicity data by considering, beyond the natural time analysis, the correlation function suggested recently in Ref. [31].

We now focus on the first aim by making use of the natural time analysis of the multiplicative cascades [3]. In a generalized Cantor set (or a multiplicative cascade), at the initial stage $M=1$ the original region is divided into K segments with possibly variable sizes, but the mass probability from the left to the right is distributed by the constant weights w_i, $i=1,2,\ldots,K$, with $\Sigma w_i = 1$. The same procedure can then be followed in each segment at the stage $M=2$, etc. Depending on whether in each stage the probability weight is distributed into the corresponding segments with a given rule, i.e., from left to right, or randomly, we discern two cases: the deterministic Cantor set (dCs) and the stochastic Cantor set (sCs), respectively. In the case of dCs with equal segments, the analysis in natural time simplifies, actually $\Pi(\omega)$ factorizes [3], which gives the advantage of an analytic calculation of the value of κ_1. For example, let us consider the initial natural time interval $\chi \in [0,1]$ and assume the weights w_i, $i=1,2,\ldots,K$ equal to the probabilities p_k. Then, following the procedure of Ref. [34], we can show that the normalized power spectrum at the stage M, i.e., $\Pi_M(\omega)$, is interconnected to that at the stage $M-1$ through the relation

$$\Pi_M(\omega) = \Pi_{M-1}\left(\frac{\omega}{K}\right) \left| \sum_{j=1}^{K} w_j \exp\left(i\omega \frac{j-1}{K}\right) \right|^2. \quad (1)$$

This factorization method of $\Pi(\omega)$ was already applied [3] to the case of two ($K=2$) segments of equal size. In this case, a single weight, let us label it $p(\equiv w_1)$, is necessary to fully define the corresponding dCs or sCs. Such a model, termed p model, was introduced [35,36] in order to describe turbulence. When studying the p model in natural time, it was found [3] that (i) for the case of the dCs we have $\kappa_1 = p(1-p)/3$ for M tending to infinity, and (ii) for the case of sCs, the most probable value of κ_1 for large values of M is approximately the value of κ_1 for the corresponding dCs, i.e.,

$$\lim_{M\to\infty} \kappa_{1,p} = \frac{p(1-p)}{3}. \quad (2)$$

Let us now apply the aforementioned results to the case of seismicity. Shuffled earthquake data are random in time and follow the GR law. The probability to observe in some area and after some waiting time an EQ of magnitude m greater or equal to m_0 is also given by $P(m \geq m_0) \propto 10^{-bm_0}$. Thus, the frequency $\nu(m)$ of EQs with magnitude m, i.e., the ones having magnitudes within $[m-\delta m, m+\delta m]$ (cf. due to the experimental errors in determining an EQ magnitude a reasonable value of δm is around 0.1) is just $\nu(m) \propto 10^{-bm}$. In light of the p model, we can now approximate the case of seismicity as follows: assuming that the largest EQ in some time interval dominates the corresponding energy release in this interval (see Fig. 1 of Ref. [34]), if an earthquake of magnitude m_0 dominates the second (segment) time interval, the first segment will be dominated by an earthquake of magnitude $m_0 - \Delta M$, having twice the frequency of m_0, i.e., $\nu(m_0 - \Delta M) = 2\nu(m_0)$. Thus, a multiplicative cascade is formed (see Fig. 1 of Ref. [34]) with a p value equal to $p = 1/(1+10^{c\Delta M})$, where c is the constant that interrelates the earthquake energy release with the magnitude, i.e., $E \propto M_0 \propto 10^{cm}$, where M_0 is the seismic moment of an EQ. Substituting the value of $\Delta M (= \frac{1}{b}\log_{10} 2)$ estimated on the basis of the GR law, we obtain $p = 1/(1+2^{c/b})$, which, in view of Eq. (2), leads to the most probable value of κ_1 given by

$$\kappa_{1,p} = \frac{2^{c/b}}{3(1+2^{c/b})^2}. \quad (3)$$

This relation interrelates $\kappa_{1,p}$ with the quantity c/b. Typical values of b and c are $b \approx 1$ and $c \approx 1.5$ [37] resulting in $\kappa_{1,p} = 0.064$. This value coincides with the value of $\kappa_{1,p}$ obtained [3] for the (randomly) shuffled earthquake data of Japan and SCEC; see also Ref. [34].

We now make use of the aforementioned Eqs. (12) and (13) of Ref. [3] and explain how they can be used for the identification of temporal correlations in real seismicity time series. These equations relate either the expectation value $E(\kappa_1)$ of κ_1 in the actually observed time series or the expectation value $E(\kappa_{1,shuf})$ of the randomly shuffled time series, when a (natural) time window of length l is sliding through the time series $Q_k \geq 0$, $k=1,2,\ldots,N$. For such a window, starting at $k=k_0$, the quantities $p_j = Q_{k_0+j-1}/\Sigma_{m=1}^{l} Q_{k_0+m-1}$ in natural time are defined and $E(\kappa_1)$ in the actually observed time series equals [3]

$$E(\kappa_1) = \kappa_{1,\mathbb{M}} + \sum_{\text{all pairs}} \frac{(j-m)^2}{l^2} \text{Cov}(p_j, p_m), \tag{4}$$

where $\kappa_{1,\mathbb{M}}$ is the value of κ_1 corresponding to the time series of the averages $\mu_j \equiv E(p_j)$ of p_j, i.e., $\kappa_{1,\mathbb{M}} = \Sigma_{j=1}^{l}(j/l)^2\mu_j - (\Sigma_{j=1}^{l}\mu_j j/l)^2$, and $\text{Cov}(p_j,p_m)$ stands for the covariance of p_j and p_m defined as $\text{Cov}(p_j,p_m) \equiv E[(p_j-\mu_j)(p_m-\mu_m)]$, while the variance of p_j is given by $\text{Var}(p_j) = E[(p_j-\mu_j)^2]$. The symbol $\Sigma_{\text{all pairs}}$ stands for $\Sigma_{j=1}^{l-1}\Sigma_{m=j+1}^{l}$. Equation (4) reveals that $E(\kappa_1)$ is determined by two factors that involve (i) the correlation of the data as reflected in the averages μ_j, e.g., due to decreasing in magnitude aftershocks in an earthquake time series and (ii) the covariances' term which sums up the correlations between all natural time lags up to $l-1$.

On the other hand, $E(\kappa_{1,shuf})$ obtained by randomly shuffling the original time series is given by [3]

$$E(\kappa_{1,shuf}) = \kappa_u\left(1 - \frac{1}{l^2}\right) - \kappa_u(l+1)\text{Var}(p) \tag{5}$$

[note that for the shuffled data $\text{Var}(p_j)$ is independent of j, and hence we merely write $\text{Var}(p) \equiv \text{Var}(p_j)$]. If Q_k do not exhibit heavy tails and have finite variance, Eq. (5) rapidly converges [3] to $E(\kappa_{1,shuf}) = \kappa_u$. Otherwise, $E(\kappa_{1,shuf})$ differs from κ_u and the difference

$$\Delta E(\kappa_{1,shuf}) \equiv \kappa_u\left(1 - \frac{1}{l^2}\right) - E(\kappa_{1,shuf}) = \kappa_u(l+1)\text{Var}(p) \tag{6}$$

provides a measure of the process' increments infinite variance. By comparing the results obtained from Eqs. (4)–(6) in a real time series, we can draw quantitative conclusions on the existence of temporal correlations even if the process' increments exhibit infinite variance.

We now turn to the existence of temporal correlations in real seismicity. As an example, we use the SCEC data with magnitude threshold $M_{thres}=2.0$ considering the area $\text{N}_{32}^{37}\text{W}_{114}^{122}$ and the period from 1981–2003. The existence of earthquake magnitude correlations in the same area was recently studied by Lippiello *et al.* [31] by calculating the correlation function $C(n)$: one divides the catalog comprised of N events in $N_L = N/L$ segments, each containing $L=125$ EQs, and define the quantity $\Delta m_j = \frac{1}{L}\Sigma_{i=jL+1}^{jL+L} m_i - \frac{1}{N}\Sigma_{i=1}^{N} m_i$ which represents the deviation of the average magnitude in the jth segment with respect to the average over the entire catalog. The quantity $C(n)$ is then given by [31]

$$C(n) = \frac{1}{n_{max}-n+1}\sum_{j=n}^{n_{max}}\sum_{i=1}^{N_L-n_{max}} \frac{\Delta m_i \Delta m_{i+j}}{(N_L - n_{max})}, \tag{7}$$

where $n_{max}=32$ the maximum "distance" between segments considered. In the absence of magnitude correlations, $C(n)$ does not depend on n and for the randomly shuffled earthquake catalog $C(n)$ fluctuates around zero exhibiting a Gaussian behavior [31]. As an example, when plotting $C(n)$ as a function of n in the case of the SCEC data along with that obtained after randomly shuffling the same catalog, these two curves [Fig. 2(a) of Ref. [34]] differ clearly. Following the reasoning of Lippiello *et al.* [31], such a difference in $C(n)$ indicates the presence of magnitude correlations in the SCEC data. This result strengthens our previous conclusion [3] that in natural time analysis the value of $\kappa_{1,p}=0.064$ of the surrogate data differs from the value of $\kappa_{1,p}=0.066$ of the original data due to the presence of temporal correlations (arising from the *ordering* of the events in natural time).

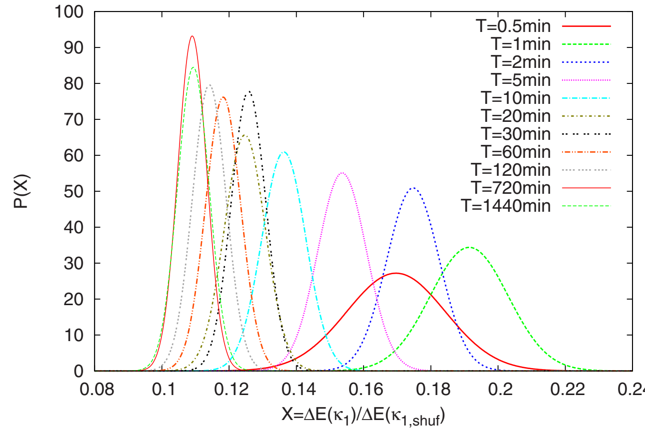

FIG. 1. (Color online) The distributions of X of Eq. (8) for various values of the maximum interoccurrence time T ranging from half a minute to one day.

The presence of temporal correlations has been further investigated upon changing the magnitude threshold M_{thres} in the SCEC catalog. A gradual decrease in the statistically *significant* temporal correlations was found [34] upon increasing M_{thres} from 2 to 3.1. This result is consistent with the expectation that upon increasing the magnitude threshold, the number of aftershocks decreases. The same trend is observed [34] when studying the values of $C(n)$. If larger values of M_{thres} are taken into account, *no definite* results can be statistically inferred.

It is well known that seismic catalogs exhibit [38] the so-called short term aftershock incompleteness (STAI). On the other hand, it has been recently shown [32] that correlations between magnitudes are larger for closer in time earthquakes. Thus, it is interesting to use natural time in a restricted catalog containing not all earthquakes but only those earthquakes at a time distance (interoccurrence time) $\delta t < T$ and choosing different values for the parameter T. This was applied to the SCEC data for $M_{thres}=2$ for values of the maximum interoccurrence time T ranging from half a minute to one day. The resulting catalogs were analyzed in natural time and the value of $E(\kappa_1)$ for the original data has been determined. Then, the same catalogs were randomly shuffled and the calculation was repeated. Following the discussion of Eqs. (4) and (6), the relative intensity of the temporal correlations with respect to the presence of process' increments infinite variance can be quantified by the ratio of the change $\Delta E(\kappa_1)$ *upon randomly shuffling* the catalog over the difference $\Delta E(\kappa_{1,shuf})$ of Eq. (6). Since our results are presented for natural time windows $l=6$ to 40, the value $\kappa_u(1-\frac{1}{l^2})$ in Eq. (6) can be substituted by its average value which is $\kappa_u^{6-40}=0.08296$. The study of these restricted catalogs showed that the distribution of

$$X \equiv \frac{\Delta E(\kappa_1)}{\Delta E(\kappa_{1,shuf})} = \frac{E(\kappa_1) - E(\kappa_{1,shuf})}{\kappa_u^{6-40} - E(\kappa_{1,shuf})} \quad (8)$$

can be approximated by Gaussian distributions (Fig. 1) differing from zero, beyond any statistical doubt, thus reflecting the existence of temporal correlations. These correlations increase (Fig. 1) as T varies from half a day to 1 min, thus agreeing with the conclusions of Ref. [32]. When T becomes less than 1 min, these correlations diminish (cf. the thick solid red curve corresponding to $T=0.5$ min with the thick dashed green curve corresponding to $T=1$ min) and this effect could be attributed to STAI: the appropriate magnitude-dependent time interval t_M to remove [38] STAI, $t_M=300 \times 10^{(M-4)/2}$ s, is for M equal to the average magnitude of these catalogs $t_{M_{ave}}=(53\pm 4)$ s.

In summary, the following three findings have emerged: first, the natural time analysis of multiplicative cascades leads to a theoretical interrelation, i.e., Eq. (3), between $\kappa_{1,p}$ of the (randomly) shuffled earthquake data and the parameter b of the Gutenberg-Richter law. This interrelation, if we just adopt a reasonable value of b, i.e., $b\approx 1$, leads to a $\kappa_{1,p}$ value that is close to 0.064 in agreement with the experimental data from SCEC and Japan. Second, upon employing natural time analysis as well as the recently introduced [31] correlation function $C(n)$, the existence of temporal correlations was investigated in real seismicity data. It was found that such correlations do exist and decrease when the magnitude threshold increases from $M_{thres}=2$ to 3.1. Third, natural time analysis leads to results that are compatible with the recent suggestion [31] that correlations between magnitudes are larger for closer in time earthquakes when the maximum interoccurrence time T varies from half a day to 1 min.

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