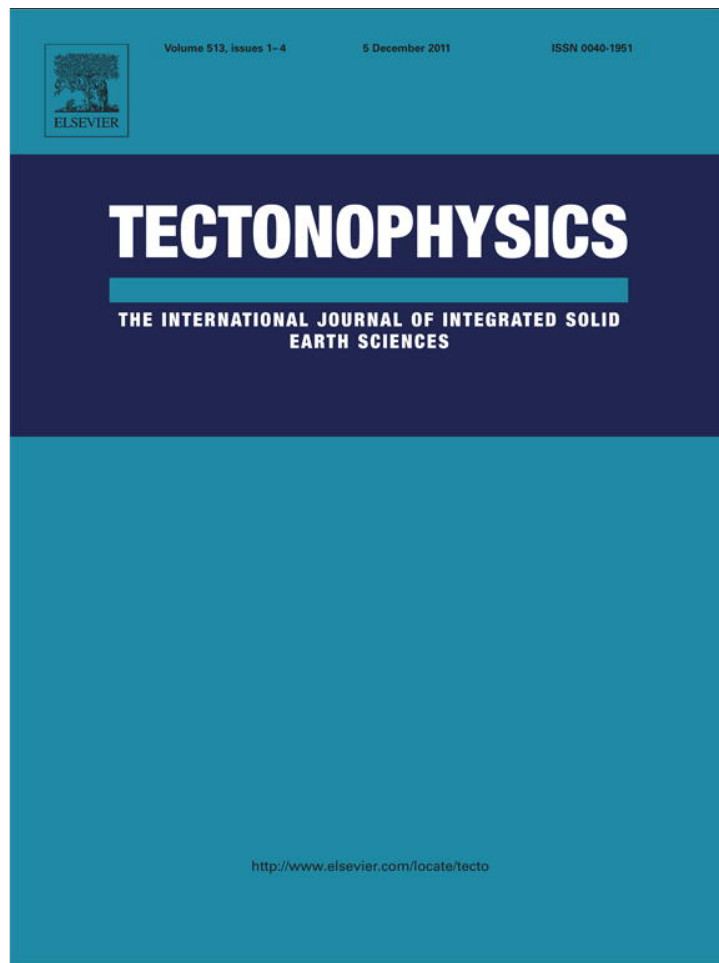


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## The change of the entropy in natural time under time-reversal in the Olami–Feder–Christensen earthquake model

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### ABSTRACT

Here, we analyze the Olami–Feder–Christensen (OFC) model for earthquakes in a new time domain, termed natural time  $\chi$ . We show that there exists a non-zero change  $\Delta S$  of the entropy in natural time upon time reversal. This reveals a breaking of the time symmetry, thus reflecting the predictability in the OFC model.

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### 1. Introduction

The self-organized criticality (SOC) concept has been originally introduced by (Bak et al., 1987) using as an example the sandpile model. The fact that avalanches seem to be uncorrelated in the original sandpile model, has been used as an argument that is not possible to predict the occurrence of large avalanches, e.g., relevant claims are cited in Ramos et al. (2006, 2009). In other words, a belief has been expressed that power-law distributed avalanches are inherently unpredictable, which came from the concept of SOC, but interpreted in the way that, at any moment, any small avalanche can eventually cascade to a large event. However, prediction is in principle possible, as it became clear from the accumulated theoretical and experimental evidence (for a review see (Varotsos et al., 2011)). For example, the prediction of extreme avalanches in self-organized critical sandpiles has been studied in recent detailed numerical studies (Garber and Kantz, 2009; Garber et al., 2009) which showed that particularly large events in a close to SOC system can be predicted on the basis of past observations.

An important contribution to SOC ideas is the Olami–Feder–Christensen (OFC) earthquake model (Olami et al., 1992) which is probably (Ramos et al., 2006) the most studied non-conservative, supposedly, SOC model. It originated by a simplification of the Burridge–Knopoff (BK) spring-block model (Burridge and Knopoff, 1967) by mapping it into a non-conservative cellular automaton simulating the earthquake's

behavior and introducing dissipation in the family of SOC systems. In the OFC model the force on a block is stored in a site of a square lattice, and the static friction threshold is assumed to have the same value over all blocks.

The criticality of the OFC model has been debated (e.g., (de Carvalho and Prado, 2000; Miller and Boulter, 2002)). Also, the SOC behavior of the model is destroyed upon introducing some small changes in the rules of the model, e.g., replacing open boundary conditions with periodic boundary conditions (Pérez et al., 1996), introducing frozen noise in the local degree of dissipation (Mousseau, 1996) or in its threshold value (János and Kertész, 1993), including lattice defects (Ceva, 1995) – which should not be confused with the intrinsic lattice defects in solids (Varotsos, 2007), e.g., Schottky (Varotsos and Alexopoulos, 1979, 1984a) or Frenkel (Varotsos, 1976; Varotsos and Alexopoulos, 1978) defects. Despite these findings as well as others which show (Peixoto and Davidsen, 2008), that it is insufficient to account for certain aspects of the spatiotemporal clustering of seismicity, the OFC model appears to show many features found in real earthquakes. As far as earthquake predictability (Pepke and Carlson, 1994) or Omori's law (Helmstetter et al., 2004; Hergarten and Neugebauer, 2002) is concerned, the OFC models appear to be closer to reality than others (Wissel and Drossel, 2006).

It has been shown (Abe et al., 2005) that the analysis of time series of complex systems in a new time domain, termed natural time  $\chi$  (Varotsos et al., 2001, 2002, 2011), reduces uncertainty and extracts signal information as much as possible. Natural time analysis (recapitulated in Section 2) enables the study of the dynamic evolution of a complex system and identifies when the system approaches the critical point. Relevant applications of this new type of analysis with interesting results have been presented in a variety of cases including

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seismicity (Sarlis et al., 2008, 2009, 2010; Varotsos et al., 2001, 2006a, 2006b, 2008, 2011), identification (Varotsos et al., 2002, 2011) of the preseismic low frequency ( $\leq 1$  Hz) variations of the electric field of the earth – termed Seismic Electric Signals (Varotsos and Alexopoulos, 1984b; Varotsos and Lazaridou, 1991; Varotsos et al., 1993) (SES) – that are emitted from the focal area when the stress reaches a critical value (Varotsos and Alexopoulos, 1984b; Varotsos et al., 1993, 2011) and precede earthquakes, e.g. for some recent examples see Uyeda and Kamogawa (2008, 2010) and Uyeda et al. (2009).

It is the main scope of this paper to show, on the basis of natural time analysis of the avalanche time series, the predictability of the OFC model whose summary is given in Section 3. To achieve this aim, we shall use recent complexity measures based on natural time, like the change  $\Delta S$  of the entropy in natural time under time reversal as discussed in Section 4. These measures have been recently found to exhibit noticeable variations when approaching the critical point in a variety of complex systems. For example, we mention that when analyzing electrocardiograms in natural time, e.g. see (Varotsos et al., 2005a), we find (Varotsos et al., 2007) that: (a) The measure  $\Delta S_l$  at the scale  $l=3$  heartbeats, identifies the sudden cardiac death risk and distinguishes sudden cardiac death individuals from truly healthy individuals as well as from those with the life-threatening congestive heart failure. (b) Among those classified as sudden cardiac death individuals, the measured  $\Delta S_l$  at the scale  $l=13$  heartbeats provides an estimate of the occurrence time of the impending risk. The main conclusion is given in Section 5.

## 2. Natural time analysis. Background

Let us consider a time-series comprising  $N$  events. For example, in the case of SES activity – which is usually a time series of dichotomous nature – each event is a pulse of given duration  $Q$ , whereas for seismicity an earthquake of magnitude  $m$ . For a time-series of avalanches each event is an avalanche of size  $s$ . The natural time  $\chi_k = k/N$  serves as an index (Varotsos et al., 2001, 2002) for the occurrence of the  $k$ -th event. In natural time analysis, the evolution of the pair  $(\chi_k, E_k)$  is considered (Varotsos et al., 2001, 2002, 2006a, 2006b), where  $E_k$  denotes the energy released during the  $k$ -th event. This energy is itself proportional to the duration  $Q_k$  of each electric pulse in the case of SES activities, whereas for seismicity it is proportional to the seismic moment  $M_0$ , which is related (Kanamori, 1978) to the magnitude  $m$  through  $M_0 \propto 10^{1.5m}$ .

The evolution of either a SES activity or a series of earthquakes in natural time, can be analyzed as follows: For a SES activity, we consider the evolution  $(\chi_k, Q_k)$ , and define the continuous function  $\Pi(\omega)$ :

$$\Pi(\omega) = |\Phi(\omega)|^2 = \left| \sum_{k=1}^N p_k \exp\left(i\omega \frac{k}{N}\right) \right|^2, \quad (1)$$

where  $p_k = Q_k / \sum_{j=1}^N Q_j$ , and  $\omega = 2\pi\phi$ ,  $\phi$  the natural frequency. For a seismic activity, we consider the evolution  $(\chi_k, (M_0)_k)$ , i.e., by ascribing to the  $k$ -th event, instead of  $Q_k$ , the corresponding seismic moment  $(M_0)_k$ . The continuous function  $\Phi(\omega)$  is then defined by Eq. (1) with  $p_k = (M_0)_k / \sum_{j=1}^N (M_0)_j$ , for seismic events. Note, that in both cases,  $\Pi(\omega)$  is a normalized power spectrum (Varotsos et al., 2001, 2002) that describes the process in natural time.

We focus on the properties of  $\Pi(\omega)$  or  $\Pi(\phi)$  for natural frequencies  $\phi$  less than 0.5, since in this range of  $\phi$ ,  $\Phi(\omega)$  or  $\Phi(\phi)$  reduces (Varotsos et al., 2001, 2002, 2011) to a characteristic function for the probability distribution  $p_k$  in the context of probability theory. According to the probability theory, the moments of a distribution and hence the distribution itself can be approximately determined once the behavior of the characteristic function of the distribution is known around zero. For  $\omega \rightarrow 0$ , a Taylor expansion of Eq.(1) reveals

that  $\Pi(\omega) \approx 1 - \kappa_1 \omega^2$  where  $\kappa_1$  is the variance of  $\chi$  given by  $\kappa_1 = \langle \chi^2 \rangle - \langle \chi \rangle^2$ , where  $f(\chi) = \sum_{k=1}^N p_k f(\chi_k)$ .

The entropy  $S$  in natural time is defined (Varotsos et al., 2011) as  $S = \langle \chi \ln \chi \rangle - \langle \chi \rangle \ln \langle \chi \rangle$ . It is dynamic entropy depending on the sequential order of pulses, e.g., see (Varotsos et al., 2005a, 2008). The entropy obtained upon considering (Varotsos et al., 2005b) the time reversal is labeled by  $S_-$ . The time reversal  $T$  is defined by  $Tp_m = p_{N-m+1}$ , which means that the last event becomes first, the last but one becomes second etc. It was found (Varotsos et al., 2005b) that, in general,  $S_-$  is different from  $S$ , and hence  $S$  shows the breaking of the time-reversal symmetry. We note that when considering a small increasing trend  $\varepsilon (> 0)$  for  $p_k$  versus  $k$ , via the parametric family (Varotsos et al., 2006b, 2008)

$$p(\chi; \varepsilon) \equiv 1 + \varepsilon(\chi - 1/2), \quad (2)$$

so that the corresponding entropy in natural time is given by

$$S(\varepsilon) \equiv \int_0^1 p(\chi; \varepsilon) \chi \ln \chi d\chi - \left[ \int_0^1 p(\chi; \varepsilon) \chi d\chi \right] \ln \left[ \int_0^1 p(\chi; \varepsilon) \chi d\chi \right], \quad (3)$$

we find (Varotsos et al., 2006b, 2008)

$$S(\varepsilon) = -\frac{1}{4} + \frac{\varepsilon}{72} - \left( \frac{1}{2} + \frac{\varepsilon}{12} \right) \ln \left( \frac{1}{2} + \frac{\varepsilon}{12} \right). \quad (4)$$

Expanding Eq. (4) around  $\varepsilon = 0$ , we obtain that

$$S(\varepsilon) = S_u + \left( \frac{6 \ln 2 - 5}{72} \right) \varepsilon + O(\varepsilon^2), \quad (5)$$

where  $S_u \equiv \frac{2 \ln 2 - 1}{4} \approx 0.0966$  is the entropy of the “uniform” distribution (Varotsos et al., 2011), e.g., when  $E_k$  are independent and identically distributed random variables. Since  $S_-(\varepsilon)$  simply equals  $S(-\varepsilon)$ , we obtain that

$$S_-(\varepsilon) - S(\varepsilon) = \left( \frac{5 - 6 \ln 2}{36} \right) \varepsilon + O(\varepsilon^3), \quad (6)$$

thus reflecting that an increasing trend in  $p(\chi; \varepsilon)$ , i.e.,  $\varepsilon > 0$ , corresponds to positive  $S_-(\varepsilon) - S(\varepsilon)$ . The difference  $S_- - S$  will be hereafter labeled  $\Delta S$  (cf. if we alternatively (Varotsos et al., 2007) use this symbol to denote the opposite quantity, i.e.,  $S - S_-$ , obviously an increasing trend would correspond to negative  $S - S_-$ ). The quantity  $\Delta S$  may also have a subscript ( $\Delta S_l$ ) meaning that the calculation is made (for each  $S$  and  $S_-$ ) at a scale  $l$  (= number of successive events). In such a procedure, a window of length  $l$  is sliding, each time by one event, through the whole time series. The entropies  $S$  and  $S_-$ , and there from their difference  $\Delta S_l$ , are calculated for each event. Thus, we form a new time series consisting of successive  $\Delta S_l$  values.

## 3. The Olami–Feder–Christensen (OFC) model

The OFC model runs as follows: we assign a continuous random variable  $z_{ij} \in (0, 1)$  to each site of a square lattice, which represents the local ‘energy’. Starting with a random initial configuration taken from a uniform distribution in the segment  $(0, 1)$ , the value  $z_{ij}$  of all sites is simultaneously increased at a uniform loading rate until a site  $ij$  reaches the threshold value  $z_{thres} = 1$  (i.e., the loading  $\Delta f$  is such that  $(z_{ij})_{max} + \Delta f = 1$ ). This site then topples which means that  $z_{ij}$  is reset to zero and an ‘energy’  $\alpha z_{ij}$  is passed to every nearest neighbor. If this causes a neighbor to exceed the threshold, the neighbor topples also, and the avalanche continues until all  $z_{kl} < 1$ . Then the uniform loading increase resumes. The number of topplings defines the size  $s$  of an avalanche or “earthquake”. This is the quantity that is used as the energy  $E_k$  in natural time analysis (Varotsos et al., 2011).

The coupling parameter  $\alpha$  can take values from zero to 0.25. Smaller  $\alpha$  means more dissipation, and  $\alpha = 0.25$  corresponds to the conservative

case. The parameter  $\alpha$  is the *only* parameter of the model, apart from the system size  $L$ , the edge length of the square lattice. Except from the initial condition the model is deterministic. The model can be supplemented by open boundary conditions in which the sites at the boundary distribute energy to the outer sites, which cannot topple, thus energy is removed at the boundary. Another possibility, is to use free boundary conditions: In this case,  $\alpha$  varies locally

$$\alpha_{ij} = \frac{1}{n_{ij} + K} \quad (7)$$

where  $n_{ij}$  is the actual number of nearest neighbors of the site  $ij$ . For sites in the bulk  $n_{ij} = 4$ , for sites at the edges  $n_{ij} = 3$  and for the four sites at the corners  $n_{ij} = 2$ . The symbol  $K$  denotes the elastic constant of the upper leaf springs measured relatively to that of the other springs between blocks (Helmstetter et al., 2004) in the BK model. Obviously the OFC model is non-conservative for  $K > 0$  for which  $\alpha_{ij} < 0.25$  in the bulk. Finally, periodic boundary conditions (PBC) can be imposed but these destroy (Pérez et al., 1996) criticality.

Concerning the study of the transient regime of the OFC model, it can be simplified (Varotsos et al., 2011) when done in terms of the quantity  $f = \sum (\Delta f)$ , which represents the total increase of  $z_{ij}$  due to the external force loading in each site. Since the loading rate is assumed uniform in time,  $f$  plays a role analogous to that of the conventional time  $T \equiv f$ . de Carvalho and Prado (2003) found that the conservative and non-conservative cases display a qualitatively different behavior. This has been also verified by natural time analysis (Varotsos et al., 2011). Here, in what follows, we shall solely focus on the stationary regime of the OFC model.

#### 4. The predictability of the OFC model using the entropy change $\Delta S$ under time reversal

We first mention that the predictability of the OFC model, for which the presence of ‘foreshocks’ (as well as ‘aftershocks’) has been identified by Hergarten and Neugebauer (2002) in the non-conservative case, has been studied by Pepke and Carlson (1994) and Pepke et al. (1994). Recently Caruso and Kantz (2011) have provided an efficient algorithm for the prediction of large avalanches solely based on the past avalanches for the non-conservative OFC model on a small world topology. Moreover, Varotsos et al. (2011) have shown that if we consider the mean ‘energy’ per site  $\zeta = \sum z_{ij} / L^2$  which is a function of the ‘time’  $T$ , as a decision variable such a goal is possible even for the OFC model on a square lattice. Here, we focus our interest on the entropy change  $\Delta S$  in natural time before large avalanches in the original OFC model on a square lattice and show that  $\Delta S$  may provide a decision variable for the prediction of a large avalanche in the next natural time step.

The occurrence of ‘foreshocks’ as well as ‘aftershocks’ in the OFC model has been exhaustively studied by Helmstetter et al. (2004). Here, we solely focus on the former (foreshocks) that are described by the so-called inverse Omori’s law (Helmstetter et al., 2003, 2004) which states that the average increase of seismicity observed at the time  $t$  before the occurrence time  $t_c$  of a mainshock is given by

$$N_f(t) = \frac{K_f}{(t_c - t + c)^{p_f}} \quad (8)$$

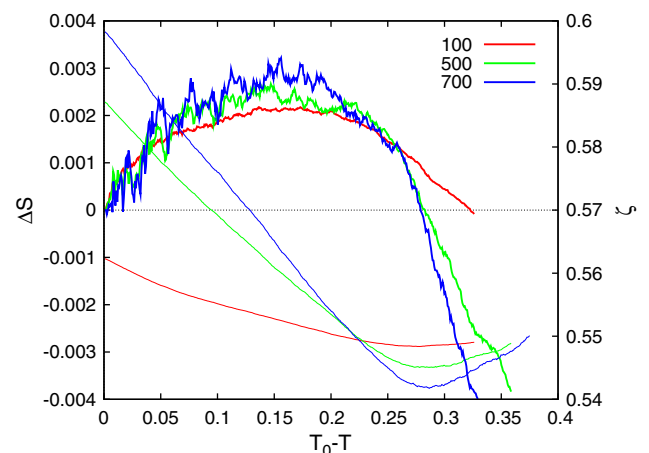
where the subscript ‘ $f$ ’ refers to the foreshocks and the quantities  $K_f$  and  $c$  are taken constants. The inverse Omori’s exponent  $p_f$  is usually close to or slightly *smaller* than the corresponding (Helmstetter and Sornette, 2003) exponent  $p_a$  of the usual Omori’s law for aftershocks. Helmstetter et al. (2004) defined ‘mainshock’ (see their definition  $d = 0$ ) as any earthquake of magnitude  $m$  which was not preceded or followed by a larger earthquake in a time window of length  $T(m)$  equal to 1% of the average return time of an earthquake of magnitude

$m$ . Foreshocks are then selected as all earthquakes occurring within the time  $T(m)$  before a mainshock. The value of  $p_f$  has been found by averaging the seismicity rate before a large number of mainshocks (Helmstetter and Sornette, 2003), because there are huge fluctuations of the rate of seismicity before individual mainshocks. Thus, following the method of study of foreshocks in Helmstetter et al. (2004), our results for  $\Delta S$  are found by averaging the values obtained before an appreciably high number of large avalanches.

Assuming that the large avalanche occurs at  $T_0$ , Fig. 1 depicts the results for the average change  $\Delta S$  (left scale) of the entropy in natural time under time reversal and the average value of the mean ‘energy’  $\zeta$  (right scale) obtained by using the last 500 avalanches (irrespective of their size) before a large avalanche of size  $s \geq 100$  (red), 500 (green) and 700 (blue) in the non-conservative OFC model with  $L = 64$  and  $K = 2$ . They have been obtained by analyzing  $10^7$  avalanches after the system has reached the steady SOC state. In the horizontal axis, the time is measured from the occurrence time  $T_0$  of the large avalanche. We find that the magnitude of  $\Delta S$  maximizes before the impending large avalanche, thus signaling the imminent major event. The positive values of  $\Delta S$  reflect, through Eq. (6), that the avalanche size tends to increase as the time approaches that of the mainshock (cf. due to the foreshocks, mentioned above, that start to become discernable from the background seismicity). The decrease of  $\Delta S$  that occurs after a maximum at  $T_0 - T \approx 0.15$  before the mainshock, reflects that the ‘effective’ trend  $\varepsilon$ , which was defined in natural time (see Eq. (2)), should simultaneously decrease. This appears so, because it is the relative intensity of the events that is considered (through  $p_k = E_k / \sum_{j=1}^N E_j$ ) in the calculations in natural time. Using Eq. (8) and taking as a measure of the ‘effective’ trend the ratio of  $p_N$  which corresponds to the last event (i.e., the foreshock that occurred just before the mainshock at time  $t_{-1}$ ) over  $p_1$ , we obtain that

$$\frac{p_N}{p_1} = \left( \frac{t_c - t_i + c}{t_c - t_{-1} + c} \right)^{p_f}, \quad (9)$$

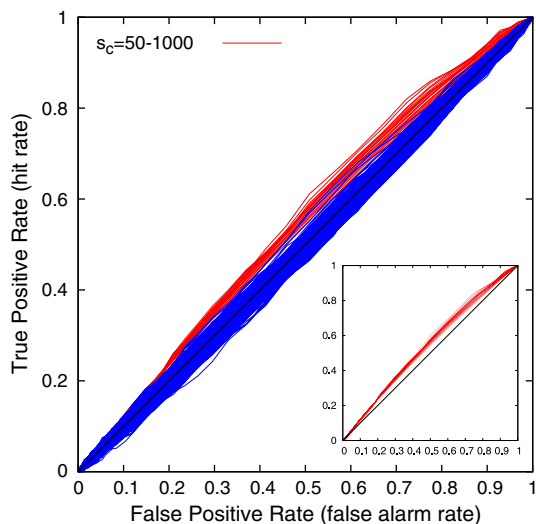
where  $t_i$  denotes the time of occurrence of the initial foreshock taken into account in the natural time entropy calculation. As  $t_i$  approaches  $t_{-1}$ , Eq. (9) results in smaller values of  $p_N/p_1$ , thus reflecting a decreasing effective trend  $\varepsilon$ . In fact, Fig. 1 shows that the average entropy change in natural time becomes zero when  $T_0 - T \approx 0$  (in this case, Eq. (9) leads to an absence of trend since  $p_N/p_1 \rightarrow 1$  as  $t_i \rightarrow t_{-1}$ ). Furthermore, note that  $\Delta S$  changes sign, becoming positive, when the



**Fig. 1.** Results from averaging the last 500 events before a large avalanche ( $s \geq 100$ , 500, 700 occurring at  $T_0$ ) in the OFC model with  $L = 64$  and  $K = 2$ : The change  $\Delta S$  (left scale, thick lines) of the entropy in natural time under time reversal and the mean energy  $\zeta$  (right scale, thin lines) as a function of the ‘time’ ( $T_0 - T$ ) to the large avalanche. Note that  $\Delta S$  maximizes before the occurrence time  $T_0$  of the large avalanche, and changes sign when  $\zeta$  almost starts to increase.

parameter  $\zeta$  almost starts to increase (cf. recall that the quantity  $\zeta$  as mentioned can be used as a predictor for the large avalanches).

Proceeding one step further, we now examine whether the entropy change  $\Delta S$  in natural time under time-reversal can be used for the prediction of a large avalanche in the next natural time step. For this reason, based on the results shown in Fig. 1, we determine the quantity  $\bar{\Delta S}$  which is the average value of  $\Delta S$  using the past events that occurred at time  $T$  within the period  $\Delta T = T_{now} - T = 0.05$  to  $0.2$ , where  $T_{now}$  stands for the present time. The quantity  $\bar{\Delta S}$  can be considered as a decision variable in the sense that the time increased probability (TIP) is turned on when  $\bar{\Delta S} > \bar{\Delta S}_c$ , where  $\bar{\Delta S}_c$  is a given threshold in the prediction. If the size  $s$  of the next avalanche is greater than a target avalanche size threshold  $s_c$ , we have a successful prediction. For binary predictions, the prediction of events becomes a classification task with two types of errors: missing an event and giving a false alarm. We therefore choose, following (Garber et al., 2009), the receiver operating characteristics (ROC) graph (Fawcett, 2006) to depict here the prediction quality. This is a plot of the hit rate versus the false alarm rate, as a function of the total rate of alarms, which here is tuned by the threshold  $\bar{\Delta S}_c$ . Only if in between the hit rate exceeds the false alarm rate, the predictor is useful. Random predictions generate equal hit and alarm rate, and hence they lead to the diagonal in the ROC plot. (If  $\bar{\Delta S}_c$  is maximum, both hit rate and false alarm rate are zero, while for very small  $\bar{\Delta S}_c$  values both rates tend to unity.) Thus, only when the points lie above this diagonal the predictor is useful. As an example, the ROC graph for  $L=64$  and  $K=2$  is shown in the inset of Fig. 2. For every given threshold value  $\bar{\Delta S}_c$  and a target threshold  $s_c$ , we get a point in this plot, thus varying  $\bar{\Delta S}_c$  we get a curve. The various curves in the middle of the inset of Fig. 2 correspond to various values of  $s_c$  increasing from the bottom to the top. Recalling that the diagonal line in such a plot corresponds to random predictions, and the points in each curve lie above it (meaningful prediction), we conclude that the precursory function  $\Delta S$  can be considered as a possible predictor. In order to investigate the statistical significance of the results as well as the reason behind the possible predictive power of  $\Delta S$ , we insert in Fig. 2 the results obtained (blue curves) from 100 trials when performing the same calculation by using randomly shuffled  $\bar{\Delta S}$  values. We observe that the results



**Fig. 2.** Receiver operating characteristics graph (red) for the OFC model with  $L=64$  and  $K=2$  when using  $\Delta S$  as a predictor: The true positive rate (hit rate) versus the false positive rate (false alarm rate) for various values of the target avalanche size threshold  $s_c = 50, 100, 150, \dots, 1000$ . At false alarm rate equal to 50% the lower curve corresponds to  $s_c = 50$  and the upper one to  $s_c = 1000$ . The blue curves depict the results obtained when repeating 100 times the same calculation using randomly shuffled  $\Delta S$  values as a predictor. The inset reproduces the red curves for the sake of clarity.

obtained for the original  $\Delta S$  time-series are well above those obtained even from the extreme of 100 trials using randomly shuffled  $\Delta S$  values. Thus, the predictions made on the basis of  $\Delta S$  are statistically significant, and as concerns the origin of their predictive power it should be attributed to the fact that  $\Delta S$  is able to catch the 'true' time arrow as mentioned.

## 5. Conclusion

The natural time analysis of the avalanches in the stationary regime of the OFC model reveals that there is a non-zero change  $\Delta S$  of the entropy  $S$  in natural time under time-reversal. This signals the breaking of the time symmetry, thus reflecting predictability in the OFC model.

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