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# Natural-time analysis of critical phenomena: The case of seismicity

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**Abstract** – We first investigate in natural time the numerical simulations of a simple deterministic self-organized critical system introduced to describe avalanches in stick-slip phenomena. It is one-dimensional and belongs to the same universality class as the train model for earthquakes introduced by Burridge and Knopoff. We show that the variance $\kappa_1 = \langle\chi^2\rangle - \langle\chi\rangle^2$ of natural time $\chi$, becomes approximately equal to 0.070 when the system approaches the critical state. Next, we analyze in natural time the small earthquakes subsequent to the low-frequency magnetic-field precursor observed near the epicenter of the Ms7.1 Loma Prieta earthquake in 1989. We find that almost five days before the mainshock, the condition $\kappa_1 \approx 0.070$ was reached.



The grand challenge to predict when earthquakes (EQs) may occur is extremely difficult. EQs exhibit [1,2] complex correlations in time, space and magnitude, on which several recent studies (*e.g.*, [3–7]) have been focused, but there is not yet [8] a comprehensive explanation of the mechanisms giving rise to their complex phenomenology. However, it is widely accepted [1,9–15] that the observed earthquake scaling laws indicate the existence of phenomena closely associated with the proximity of the system to a critical point. Within the frame of this widespread belief, an order parameter of seismicity has been suggested [16] on the basis of natural time (see below). Studying the probability density function of this parameter, it was demonstrated [16] that the seismicities in various regions (as well as the worldwide seismicity) give rise to probability density function curves that collapse on the same master (universal) curve. In addition, the study of the order parameter fluctuations, relative to the standard deviation of its distribution, led to the conclusion [16] that the scaled distributions collapse on the same curve, which exhibits features similar to those in several equilibrium and nonequilibrium critical phenomena [17,18] as well as in non-stationary biological signals including heart rate [19], locomotor activity [20] etc.

Very recently [21] we made use of the aforementioned order parameter of seismicity, hereafter labelled $\kappa_1$ (see below), together with the detrended fluctuation analysis of the magnitude time series [6] to investigate the period before and after a significant mainshock. The study was focused on two significant EQs that occurred in California in 1992 and 1999, *i.e.*, the Landers and the Hector Mine earhquakes. Magnitude time series for various lengths of $W$ EQs that occurred before or after the mainshock have been considered. The natural-time analysis of these time series, revealed that "foreshocks" exhibit a behavior characteristic of systems close to their critical point: *i.e.*, the probability distribution function $P(\kappa_1)$ *vs.* $\kappa_1$ exhibits a bimodal feature. In an attempt to quantify this feature, we considered a measure of the variability of $\kappa_1$, which was then used as decision variable for the "prediction" of the occurrence of a large earthquake in the next time step based solely on the magnitudes of previous earthquakes. The results were found to convincingly outperform chance. However, they were not spectacular and this was attributed to the following possibility: When using a constant $W$, it may not correspond to the time at which the focal area of the impending mainshock enters into the critical regime which may be captured however by the detection of precursory electric signals, termed Seismic Electric Signals (SES, see below) *if* such measurements are available. Here, we investigate this challenging

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possibility. In particular, we analyze in natural time the small events (earthquakes) that occur after a SES detection and precede a mainshock. This will be investigated here for the 18 October 1989 Ms7.1 Loma Prieta earthquake (California) for which precursory SES like changes have been reported, as it will be explained below. Before proceeding, however, to this analysis of real seismic data, we also investigate what happens with the aforementioned order parameter of seismicity in a well-known earthquake model.

An EQ is a stick-slip dynamical instability of a pre-existing fault driven by the motion of a tectonic plate [2,22]. A relatively simple dynamical model that contains much of the essential physics of earthquake faults is the so-called spring-block model originally proposed [9] by Burridge and Knopoff. It consists of an assembly of blocks, each of which is connected via elastic springs to the nearest-neighboring blocks. The blocks are also connected to the driving plate by elastic springs and rest on a surface with a velocity-weakening stick-slip friction force (the friction force decreases as the velocity is increased). When the force acting on a block overcomes the static friction with the surface, the block slips. Then a redistribution of forces takes place in the neighbors that eventually trigger new displacements. An earthquake event is defined as a cluster of blocks that move (slip) due to the initial slip of a single block. A numerical study in one dimension was already made by Burridge and Knopoff [9] and later Carlson, Langer and others [10,23] proceeded to more extensive studies of the one-dimensional and two-dimensional Burridge-Knopoff models focusing on the magnitude distribution of earthquake events. Recently, spatiotemporal correlations of the two-dimensional Burridge-Knopoff model have been studied [24] by considering also long-range inter-block interactions. We shall show that, if the events in the Burridge-Knopoff model are analyzed in terms of a new time domain, termed natural time $\chi$, proposed recently [25], the recognition [25–29] of the system entering the critical stage can be achieved. We clarify that natural time has been shown [30], upon employing the Wigner function and the generalized entropic measure proposed by Tsallis [31], to be optimal in the time-frequency space which conforms to the desire to reduce uncertainty and extract signal information as much as possible.

In analyzing a time series comprising $N$ events, we define an index for the occurrence of the $k$-th event by $\chi_k = k/N$, which we term natural time. We then study the evolution of the pair $(\chi_k, Q_k)$, where $Q_k$ stands for the energy of the $k$-th event, by using the normalized power spectrum

$$\Pi(\omega) = |\Phi(\omega)|^2, \tag{1}$$

defined by

$$\Phi(\omega) = \sum_{k=1}^{N} p_k \exp\left(i\omega \frac{k}{N}\right), \tag{2}$$

where $p_k = Q_k / \sum_{n=1}^{N} Q_n$ represents the normalized size of the $k$-th event and $\omega$ stands for the angular natural frequency. The quantities $p_k$ can be considered as probabilities since they are positive and additive to unity. A quantity $\kappa_1$ is derived by the Taylor expansion $\Pi(\omega) = 1 - \kappa_1\omega^2 + \kappa_2\omega^4 + \cdots$, where

$$\kappa_1 = \sum_{k=1}^{N} p_k \chi_k^2 - \left(\sum_{k=1}^{N} p_k \chi_k\right) \equiv \langle\chi^2\rangle - \langle\chi\rangle^2. \tag{3}$$

This quantity, which is formally the variance of natural time $\chi_k$ weighted for $p_k$, quantifying the dispersion of the most prominent events within the "rescaled" interval $(0, 1]$, varies upon the occurrence of any new event. It has been demonstrated that this analysis enables recognition of the complex dynamic system under study entering the critical stage [25–29], namely this occurs when the variance $\kappa_1$ becomes approximately equal to 0.070. Originally the condition $\kappa_1 = 0.070$ for the approach to criticality was theoretically derived [25] for the Seismic Electric Signals (SES), which are transient low-frequency ($\leqslant 1$ Hz) electric signals that have been repeatedly observed before EQs [28]. SES are emitted from earthquake focal zones when the seismogenic stress reaches some critical level [28] (see below). We will investigate here whether the same condition $\kappa_1 = 0.070$ holds for a simple version of the Burridge-Knopoff model.

In the Burridge-Knopoff model studied by Carlson, Langer and others, each block is connected, as mentioned above, to the driving element. To model the dynamics of earthquakes, Burridge and Knopoff in their original work [9] also studied the case of a chain of blocks (situated on a rough surface with friction) connected by elastic springs and pulled only at one end with a constant small velocity. The dynamics of the model is as follows: All the blocks are initially at rest. As the driver pulls the first block, the latter remains stuck until the elastic force overcomes the static friction. When this occurs, the first block will move a little. Such small events (or earthquakes) will continue and increase the elastic force on the second block. When the elastic force on the second block overcomes the friction force, an event involving the two blocks will occur. The dynamics continues with events involving three, four, five or all the blocks in the system. This model is usually called the "train model" since it has some similarity with a train, where the driving force is applied only at one end of the chain (*e.g.*, [32]). The dynamics here is governed by coupled ordinary differential equations which makes its study very time consuming. To make this system more amenable to computer simulations, de Sousa Vieira [33] introduced a continuous cellular automaton that exhibits Self-Organized Criticality (SOC), pioneered by Bak *et al.* [34], and belongs to the same universality class as the train model. This deterministic one-dimensional model for the avalanches in stick-slip phenomena, which is very close to the case of an array of connected pendulums first discussed by Bak *et al.* [34], is defined as follows (see [33,35,36]): Consider a one-dimensional system, where a continuous (force) variable $f_l \geqslant 0$ is associated with each site $l$, $l = 1, 2, \ldots, L$. Initially all $f_l$ have the same value $f_0$ which lies below a threshold $f_{th}$. One can set $f_{th} = 1.0$ without loss of generality. The basic time step consists of varying the force on the first site according to $f_1 = f_{th} + \delta f$; the system then relaxes with a conservative redistribution of the forces at the site $f_l \geqslant f_{th}$ (toppling site) and its nearest neighbors according to $f_l = \Psi(f_l - f_{th})$ and $f_{l\pm1} = f_{l\pm1} + \Delta f/2$, where $\Delta f$ is the change of force at the overcritical site and $\Psi(x)$ a periodic nonlinear function. This condition mimics the redistribution of forces when the block $l$ is displaced (stick-slips) by $\Delta x_l$ during an "earthquake" in the train model [33]. The relaxation continues until all sites have $f_l < f_{th}$ for all $l$ . The size of the 'earthquake' corresponds to the number of topplings, $s$, required for the system to relax, and is considered here as the appropriate value of $Q_k$ in natural time. Then, the driving force at the first site sets in again. This is complemented by open boundary conditions; *i.e.*, the force is "lost" at $l = 1$ and $l = L$. The nonlinear periodic function used here (which means that, when considering that the force supposed here mimics the net effect of the two forces in the train model, *i.e.*, the elastic and the friction forces, the periodicity of the elastic force dominates over the form of the friction force) is similar to the one used in refs. [33,35], *i.e.*, a sawtooth function $\Psi(x) = 1 - ax + [ax]$, where $[\ldots]$ denotes the integer part of $ax$ and $a$ is a number. It was shown [33] that such a system evolves to a SOC state where the avalanche distributions are scale free limited only by the overall system size.

In fig. 1, we present the results obtained from this model using the same parameters as in ref. [35], *i.e.*, $L = 1024$, $a = 4$, $f_0 = 0.87$ and $\delta f = 0.1$. In fig. 1(a), the number of topplings $s$ is plotted in red *vs.* the avalanche number $i$ for the first 160000 avalanches which shows in fact how these series of avalanches can be read in natural time. The blue curve in fig. 1(a), shows how the quantity $\kappa_1$ evolves avalanche by avalanche. There, we also plot in green the total force $X(i)$ of the system after each avalanche, computed from $X(i) = \sum_{l=1}^{L} f_l(i)$, whose stabilization provides a measure of the approach to SOC [35]. An inspection of fig. 1(a) reveals that (after the transient and hence) when the system enters into the critical state, the $\kappa_1$ value fluctuates around 0.070 (designated by the thick blue line). The latter becomes clear in fig. 1(b), which reproduces fig. 1(a) but in an enlarged time scale for the first 40000 avalanches and shows that for $i > 5000$ (*i.e.*, just when the system enters into the SOC state) $\kappa_1$ scatters around 0.070. This behavior has been verified for a wide range of parameters $L$, $a$, $f_0$ and $\delta f$ just before the SOC state is reached. Note that once the statistically steady SOC state is established, the $\kappa_1$ value gradually increases reaching the corresponding value of $\kappa_u = 1/12$ of a "uniform" distribution (see ref. [26]). (This could be seen, for example, in fig. 1(a) if it is extended to $10^6$ avalanches.) The model discussed here leads to a power law with a realistic $b$-value of the Gutenberg-Richter law. In particular, de Sousa Vieira [33] concluded that the distribution of avalanche sizes $s$ is a power law with an exponent $\tau \approx 1.54$ that corresponds to $b \approx 0.81$. This lies in the range (0.8 to 1.2) of the $b$-values found experimentally [37]. In spite of this agreement, however, the Burridge-Knopoff model cannot account for the observed spatiotemporal complexity of seismicity, *e.g.* Omori's law for aftershocks [24].

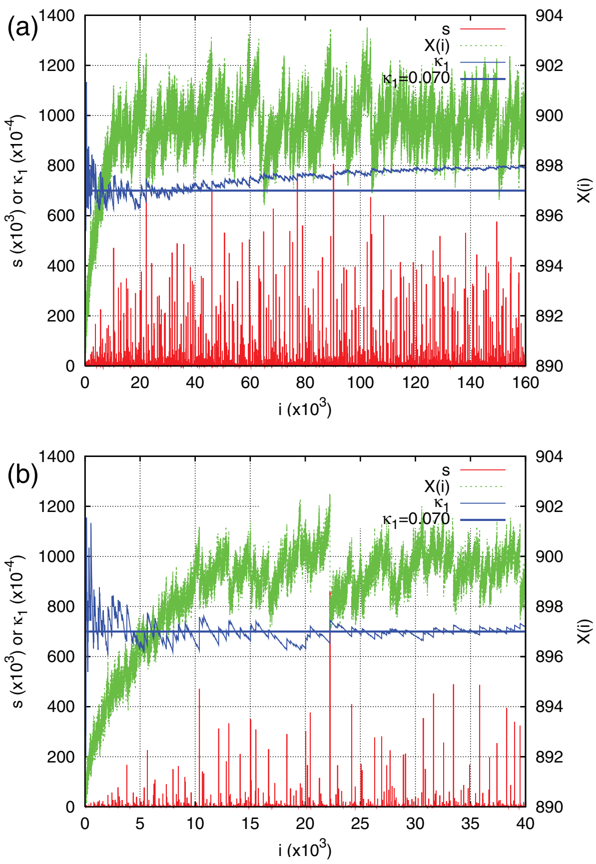


Fig. 1: (Color online) Results of the model discussed in the text for the first, $i = 1$ to $i = 160000$ (a), or to $i = 40000$ (b), avalanches obtained by numerical modelling as read in natural-time domain, for $L = 1024$, $a = 4$, $f_0 = 0.87$ and $\delta f = 0.1$: the avalanche size $s$ (red impulses, left axis), the variance $\kappa_1$ (blue lines, left axis) and the total force of the system after each avalanche $X(i)$ (green dotted lines, right axis) are plotted *vs.* the number of avalanches $i$. Panel (b) is an excerpt of (a) and shows the approach of $\kappa_1$ to $\kappa_1 = 0.070$ (thick horizontal blue line) as the system approaches SOC. (For an extension of this figure to $10^6$ avalanches, see the text.)

In the focal region of a future EQ the stress gradually changes *before* failure. In that region containing ionic materials with aliovalent impurities, extrinsic defects are present due to charge compensation. These defects are attracted by nearby aliovalent impurities, thus forming electric dipoles that can change their orientation in space through defect migration [28]. Stress variations affect the

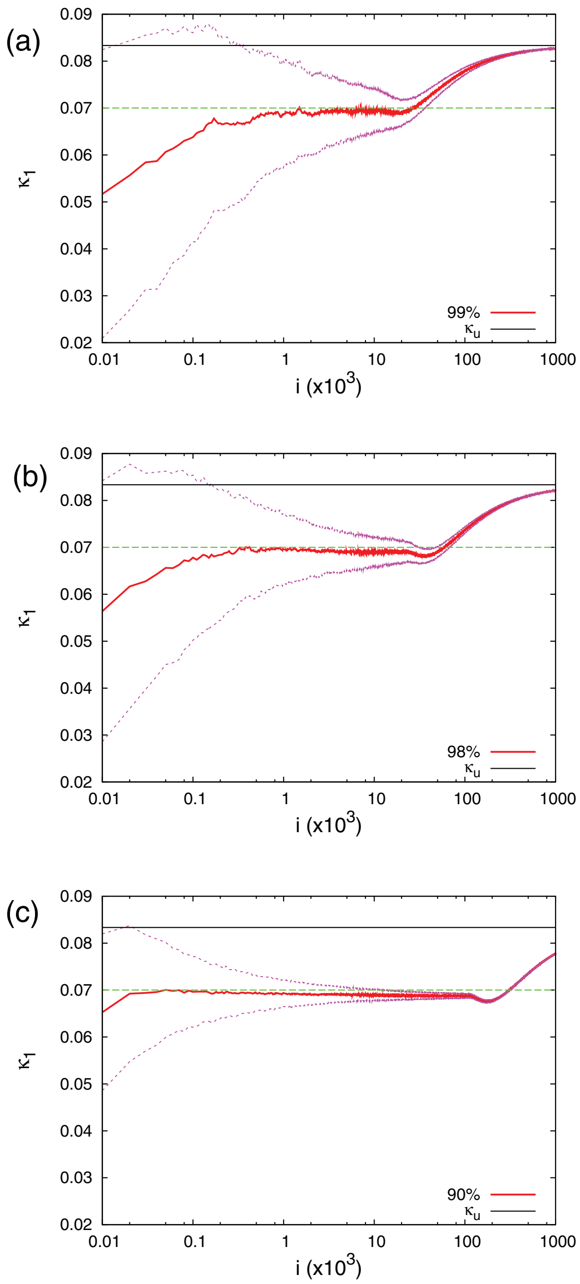


Fig. 2: (Color online) The average value of $\kappa_1$ (thick solid red line), together with the one standard deviation ($\pm\sigma$) interval (magenta dotted lines), when decreasing $\bar{f}_l$ to (a) 99%, (b) 98% and (c) 90% of its value at SOC *vs.* the number of avalanches $i$ taken into account in the natural-time calculation. These values have been obtained by numerical modelling as follows: We considered $10^3$ systems with the initial $f_l$ values randomly scattered around $f_0 = 0.87$. Each system was driven to SOC and in order to obtain a reliable series ($f_{l_{SOC}}$), $l = 1, 2, \ldots, L$, the first $10^7$ avalanches were ignored in natural-time analysis. Then, each of these $f_l$ values was reduced to 99%, 98% and 90%, respectively, of its value at SOC, *i.e.*, $f_{l_{SOC}}$, and natural-time analysis was initiated ($i = 0$). The horizontal dashed green line corresponds to $\kappa_1 = 0.070$, while the thick black solid one to $\kappa_u = 1/12$. We observe that in all cases, $\kappa_1$ approaches the critical value ($\kappa_1 = 0.070$) as the number of avalanches $i$ increases, and the system returns to SOC.

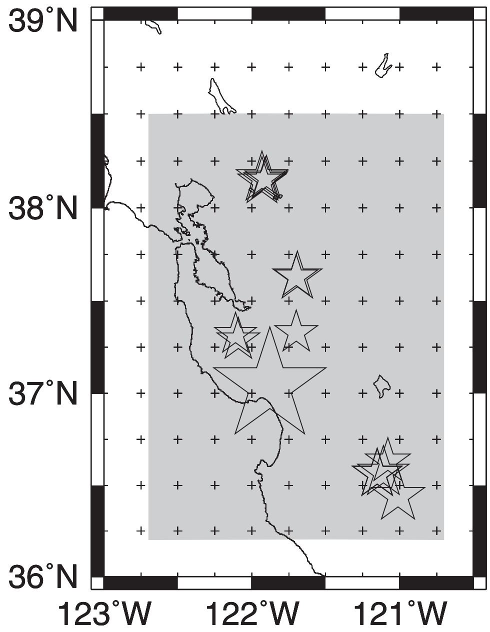


Fig. 3: A map of the area $\mathrm{N}_{36.2}^{38.5}\mathrm{W}_{122.7}^{120.7}$ (shaded) surrounding the epicenter of the Loma Prieta earthquake (large star) in which the seismicity after the initiation on 12 September 1989 of the precursory magnetic-field variations is analyzed in natural time.

thermodynamic parameters of this migration, thus may result in gradual decrease of the relaxation time of these dipoles. When the stress reaches a *critical* value [28], a *cooperative* orientation of these electric dipoles takes place that reflects the emission of a transient electric signal, which constitutes the SES, and later the failure occurs. It is commonly accepted that, after the mainshock occurrence, the stress value reduces to a smaller value, a fact however which is not fully captured by the simple Burridge-Knopoff model considered here. In other words, in the steady SOC state of this model the system has an average $f_l$ value, $\bar{f}_l$, around $\bar{f}_l = 0.8785$ that remains almost constant (*i.e.*, practically within 0.0055) after the occurrence of any avalanche (cf. $X(i)$ in fig. 1). Our computations reveal (see fig. 2) that when considering a reasonable decrease, *e.g.*, by a few percent, of $\bar{f}_l$, the system exits the steady SOC state and then returns to it through a transient in which $\kappa_1$ value scatters around 0.070, similar to that depicted in fig. 1. Hence, the value $\kappa_1 = 0.070$ can be considered as *quantifying the extent of the organization of the complex system at the onset of the critical stage*. We emphasize that such a behavior is not observed for a variant of the model which *does not* exhibit SOC [33], *e.g.*, when using, instead of a periodic function $\Psi(x)$, the strictly non-increasing function introduced by Nakanishi [38].

To examine whether the aforementioned condition $\kappa_1 \approx 0.070$ is applicable to real earthquakes, we now consider, as mentioned, the example of the 18 October 1989 Loma Prieta earthquake. To the best of our knowledge, this is the

Table 1: The seismic data analyzed in natural time. The magnitude $M$ corresponds either to $M_L$ or $M$d reported from the Northern California Earthquake Data Center (`http://www.ncedc.org/ncedc/catalog-search.html`) available on 8 January 2010. This is converted to seismic moment according to $\log_{10}M_0 = 1.5M_L + \text{const.}$

| Number | Magnitude $M$ | Date | Time (UT) | Lattitude | Longitude |
|---|---|---|---|---|---|
| 1 | 2.7 | 1989/9/16 | 18:41:24 | 37.33 | −121.70 |
| 2 | 3.2 | 1989/9/28 | 15:42:37 | 36.57 | −121.11 |
| 3 | 2.7 | 1989/10/1 | 12:21:37 | 38.15 | −121.90 |
| 4 | 3.0 | 1989/10/1 | 13:10:24 | 38.14 | −121.93 |
| 5 | 3.2 | 1989/10/1 | 13:19:27 | 38.16 | −121.93 |
| 6 | 3.1 | 1989/10/1 | 22:08:35 | 36.56 | −121.15 |
| 7 | 3.1 | 1989/10/1 | 22:09:17 | 36.56 | −121.15 |
| 8 | 2.7 | 1989/10/2 | 11:20:19 | 38.15 | −121.91 |
| 9 | 2.6 | 1989/10/6 | 15:53:36 | 37.32 | −122.11 |
| 10 | 3.3 | 1989/10/8 | 12:36:46 | 36.44 | −121.01 |
| 11 | 2.7 | 1989/10/9 | 11:51:24 | 37.63 | −121.70 |
| 12 | 2.7 | 1989/10/9 | 12:06:02 | 37.29 | −122.09 |
| 13 | 3.1 | 1989/10/9 | 12:42:03 | 37.63 | −121.69 |
| 14 | 2.8 | 1989/10/13 | 12:22:11 | 36.63 | −121.08 |
| 15 | 7.0 | 1989/10/18 | 00:04:15 | 37.04 | −121.88 |

most well-known case in USA for which clear precursory electromagnetic variations were reported. Almost one month before this earthquake, *i.e.*, on 12 September 1989, anomalous magnetic-field variations were recorded at a site just 7 km from the earthquake epicenter [39,40]. These are strikingly similar to the magnetic-field variations that accompany the SES activities observed in Greece for earthquakes with magnitude 6.5 or larger [41].

We now analyze in natural time all the events (earthquakes) that occurred after 12 September 1989, which is the date of the initiation of the aforementioned (SES like) precursory magnetic-field change, within the area $N^{38.5}_{36.2}W^{120.7}_{122.7}$ (hereafter labelled A, shaded in fig. 3) surrounding the earthquake epicenter. The seismic data used here (see table 1) are from the Northern California Earthquake Data Center and the relevant epicenters are depicted in fig. 3 (small stars). We set the natural time to zero at the initiation time of the magnetic change, and then formed time series for the area $A$ each time a small earthquake (with magnitude $M$ exceeding a certain threshold $M_{thres}$, *i.e.*, $M \geqslant M_{thres}$) occurred, *i.e.*, when the number of the events is increased by one. The quantity $\kappa_1$ for each of the time series was computed for the pairs $(\chi_k, Q_k)$. The quantity $Q_k$ was taken as the seismic moment $M_{0k}$ of the $k$-th event, since $M_0$ is roughly proportional to the energy released during an earthquake calculated from the relation $\log_{10}M_0 \approx 1.5M_L + \text{constant}$ (H. Kanamori, personal communication). Note, however, that when the area $A$ reaches criticality, one expects in general, from spatial invariance of criticality, that all subareas also reach criticality simultaneously. Thus, in order to check whether criticality has been approached at the occurrence of a new event $k$ within the area $A$, we construct all the possible subareas of $A_{M_{thres}}$ that necessarily include the event $k$ and examine if their $\kappa_1$ values

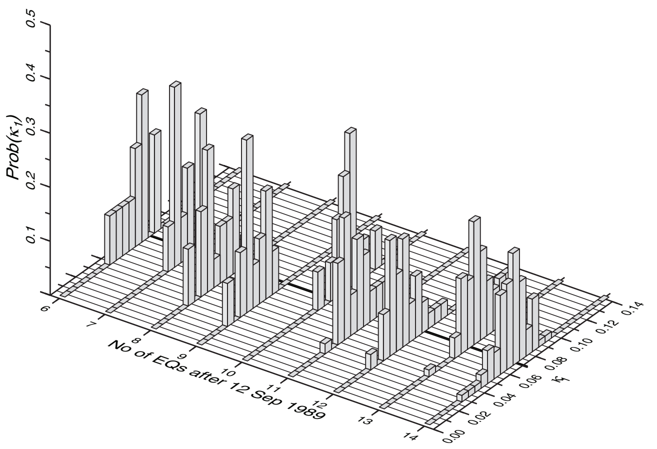


Fig. 4: A three-dimensional plot of Prob($\kappa_1$), $z$-axis, *vs.* $\kappa_1$ ($y$-axis) for the seismicity as it evolves event by event ($x$-axis) in the area $N^{38.5}_{36.2}W^{120.7}_{122.7}$, for $M_{thres} = 2.6$, subsequent to the initiation on 12 September 1989 of the precursory (SES like) magnetic-field variations. The last event (No. 14), for which Prob($\kappa_1$) maximizes at $\kappa_1 = 0.070$ (thick black line), corresponds to the magnitude 2.8 earthquake that occurred at 12:22 UT on 13 October 1989 with an epicenter at 36.63° N 121.08° W.

reveal a probability distribution Prob($\kappa_1$) maximized at 0.070 (see ref. [42] for more details). Following Davidsen *et al.* [43], we considered only earthquakes with $M > 2.5$ in order to have homogeneous and complete catalog. In other words, we take $M_{thres} = 2.6$. The results are depicted in fig. 4, which shows how Prob($\kappa_1$) *vs.* $\kappa_1$ evolves upon the occurrence of each event before the 18 October 1989, Ms7.1 Loma Prieta earthquake. We see that Prob($\kappa_1$) maximizes at $\kappa_1 = 0.070$ upon the occurrence of a 2.8 event at 12:22 UT on 13 October 1989, *i.e.*, almost 5 days *before* the mainshock. This calculation considers of course only the $M_{0k}$ values of the events that occurred until 12:22 UT on 13 October 1989. The same behavior is found when repeating the calculation for larger-magnitude thresholds, *i.e.*, $M_{thres} = 2.7$ and 2.8, showing that the maximum of Prob($\kappa_1$) *vs.* $\kappa_1$ is again observed at $\kappa_1 = 0.070$ on 13 October 1989.

To summarize: First, natural-time analysis was made for a one-dimensional self-organized criticality model introduced to describe avalanches in stik-slip phenomena. It belongs to the same universality class as the train model for earthquakes that was originally suggested by Burridge and Knopoff [9]. We found that the condition $\kappa_1 = 0.070$ is obeyed when the system acquires criticality immediately after the transient regime. Second, as an example to examine whether this condition is applicable to real earthquakes, we analyzed in natural time the seismicity *before* the 18 October 1989, Ms7.1 Loma Prieta earthquake. It was found that, in the area surrounding the epicenter of the Loma Prieta earthquake, the small earthquakes that occurred after the initiation on 12 September 1989 of the precursory (SES like) magnetic-field variations [39,40] attained criticality five days before the occurrence of the mainshock. It was also found that this condition held exhibiting spatial as well as magnitude threshold invariance.

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