Nonextensivity and natural time: The case of seismicity

N. V. Sarlis, E. S. Skordas, and P. A. Varotsos*
Solid State Section and Solid Earth Physics Institute, Physics Department, University of Athens, Panepistimiopolis, Zografos, Athens 157 84, Greece

(Received 18 April 2010; revised manuscript received 14 June 2010; published 11 August 2010)

Nonextensive statistical mechanics, pioneered by Tsallis, has recently achieved a generalization of the Gutenberg-Richter law for earthquakes. This remarkable generalization is combined here with natural time analysis, which enables the distinction of two origins of self-similarity, i.e., the process' memory and the process' increments *infinite* variance. By using also detrended fluctuation analysis for the detection of long-range temporal correlations, we demonstrate the existence of both temporal and magnitude correlations in real seismic data of California and Japan. Natural time analysis reveals that the nonextensivity parameter q, in contrast to some published claims, cannot be considered as a measure of temporal organization, but the Tsallis formulation does achieve a satisfactory description of real seismic data for Japan for $q=1.66$ when supplemented by long-range temporal correlations.

DOI: 10.1103/PhysRevE.82.021110 PACS number(s): 05.90.+m, 89.75.Da, 91.30.Ab, 05.45.Tp

I. INTRODUCTION

It is well known that seismicity exhibits power laws chief among of which are the following three: first, the Gutenberg-Richter (G-R) distribution

$$P(E) \sim E^{-\gamma} \quad (\text{with } \gamma \approx 5/3) \tag{1}$$

of earthquake (EQ) energies E, which alternatively states that the (cumulative) number of earthquakes with magnitude greater than m occurring in a specified area and time is given by

$$N(>m) \sim 10^{-bm}, \tag{2}$$

where b is a constant, which varies only slightly from region to region [cf. Eq. (2) holds both *regionally* and *globally*] being generally in the range $0.8 \leq b \leq 1.2$ (see Ref. [1] and references therein). Considering that the seismic energy E released during an earthquake is related [2] to the magnitude through $E \sim 10^{cm}$, where c is around 1.5, Eq. (2) turns to Eq. (2), where $\gamma=1+b/1.5$. Hence, $b \approx 1$ means that the exponent γ is around $\gamma=1.6-1.7$. Second, the Omori law describes the temporal decay of aftershock activity and its modified form [3] (see also Ref. [4]) is given by the relation

$$r(t,m) = \frac{1}{\tau_0[1 + t/c(m)]^p}, \tag{3}$$

where $r(t,m)$ is the rate of occurrence of aftershocks with magnitudes greater than m per day, t is the time that has elapsed since the mainshock, and τ_0 and $c(m)$ are characteristic times. Note that $p \approx 1$ for large earthquakes (e.g., see Ref. [5]). Third, the Båth law [6] according to which the difference in magnitude Δm between a mainshock and its largest detected aftershock is approximately a constant independent of the mainshock magnitude, typically $\Delta m \approx 1.2$. These three laws have been incorporated in Ref. [4] to give a generalized Omori law for aftershock decay rates that depend on several parameters specific for each given seismogenic region. There exist additional power laws referring to the distribution $\sim 1/L^2$ of fault lengths L [7], the fractal structure of fault networks [8], as well as the universal law for the distribution of waiting times and seismic rates derived by Bak *et al.* [9] from the analysis of space-time windows. It is widely accepted [10–12] that these earthquake scaling laws indicate the existence of phenomena closely associated with the proximity of the system to a critical point [13]. Deviations from these scaling laws have been observed and their explanation have also attracted a great interest (e.g., see Ref. [14] and references therein). Despite the intense efforts, however, the mechanism behind the complex spatiotemporal behavior of earthquakes still remains a major challenge [15,16].

Nonextensive statistical mechanics [17,18], pioneered by Tsallis [19], provides a consistent theoretical framework for the studies of complex systems in their nonequilibrium stationary states, systems with (multi)fractal and self-similar structures, long-range interacting systems, etc. This framework offered recently a generalization of the G-R law (see Sec. II for details). It is the main object of the present study to show that this nonextensive G-R generalization does enable an improved study of the observed seismic data fluctuations. To achieve this goal we combine this G-R generalization with another time domain, termed natural time [20,21], which has been shown to reveal novel dynamic features hidden behind the time series of complex systems in diverse fields (e.g., earth sciences [21–24], biology [21], electrocardiograms [25], and physics [26]). This time domain (see Fig. 1 that will be discussed later), when employing the Wigner function [27] and the generalized entropic measure proposed by Tsallis [19], has been demonstrated [28] to be optimal for enhancing the signal's localization in the time-frequency space [29], which conforms to the desire to reduce uncertainty and extract signal information as much as possible. Natural time analysis enables the study of the dynamic evolution of a complex system and identifies when the system approaches the critical point. This occurs when the value of the variance of natural time $\langle\chi^2\rangle - \langle\chi\rangle^2$ $(\equiv \kappa_1)$ (see Sec. III) becomes equal [21–24] to 0.070. In addition, natural time enables the distinction [30] of the two origins of self-

*Corresponding author; pvaro@otenet.gr

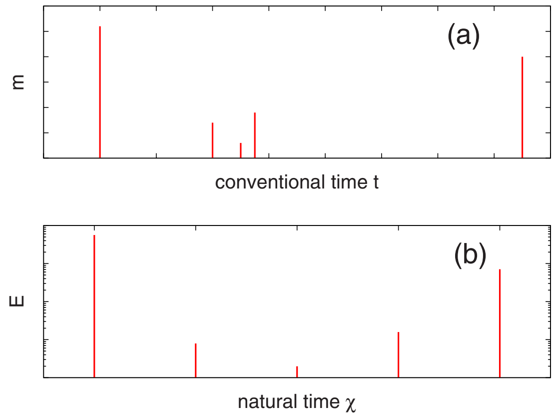

FIG. 1. (Color online) How a series of earthquakes in (a) conventional time is read in (b) natural time. The symbols m and E stand for the magnitude and the energy in the upper and lower panels, respectively.

similarity observed in signals emitted from complex systems as it will be further discussed below.

Complex systems usually exhibit irregular behavior which at first looks erratic, but in fact possesses scale-invariant structure (e.g., see Ref. [31]). A process $\{X(t)\}_{t\geq 0}$ is called self-similar [32] if, for some $H>0$,

$$X(\alpha t) = \alpha^H X(t) \quad \forall\, \alpha > 0, \tag{4}$$

where the symbol of equality refers here to all finite-dimensional distributions of the process on the left and right, and the parameter H is called the self-similarity index or exponent. Equation (4) means a "scale invariance" of the finite-dimensional distributions of $X(t)$, which does *not* imply, in stochastic processes, the same for the sample paths (e.g., see Ref. [33]). Examples of self-similar processes are Brownian, fractional Brownian motion (fBm), and Lévy stable and fractional Lévy stable motion (fLsm). Lévy stable distributions (which are followed by many natural processes, e.g., [34]) differ greatly from the Gaussian ones because they have heavy tails and their variance is infinite (e.g., [33,35]).

When analyzing data from systems that exhibit scale-invariant structure the following important question raises: in several systems this nontrivial structure points to long-range *temporal* correlations; in other words, the self-similarity results from the process memory *only*. This is the case, for example, of fBm or of seismic electric signal (SES) activities. The latter are transient low-frequency (≤ 1 Hz) electric signals emitted before earthquakes [36] presumably as follows [37,38]: beyond the usual intrinsic lattice defects [39] in ionic solids doped with aliovalent impurities, extrinsic defects are formed that are attracted by the nearby impurities, thus forming electric dipoles the relaxation time of which may decrease in the focal area of an impending earthquake as the stress gradually increases. When the stress (pressure) reaches a *critical* value, a *cooperative* orientation of these dipoles occurs, which leads to the emission of a transient signal that constitutes the SES activity characterized by critical dynamics (infinitely ranged temporal correlations [21,23]). Alternatively, the self-similarity may solely result from the process' increments "infinite" variance, e.g., Lévy stable motion (cf. but extreme events have to be truncated for physical reasons [40]). In general, however, the self-similarity may result from both these origins (e.g., fLsm). In Refs. [30,41] it was discussed in detail (see also Sec. III) how a distinction of the two origins of self-similarity (i.e., process' memory and process' increments infinite variance) can be in principle achieved by employing the natural time analysis.

In order to quantify the long-range temporal correlations, here we make use of the detrended fluctuation analysis (DFA) [42,43], which has been established as a robust method suitable for detecting long-range power-law correlations embedded in nonstationary signals (for recent applications on DFA see Ref. [44]). This can be summarized as follows. We first sum up the original time series and determine the profile $y(i)$, with $i=1,\ldots,N$. We then divide this profile of length N into N/l ($\equiv N_l$) nonoverlapping fragments of l observations. Next, we define the detrended process $y_{l,\nu}(m)$, in the νth fragment, as the difference between the original values of the profile and the local linear trend. We then calculate the mean variance of the detrended process: $F^2(l)=(1/N_l)\Sigma_{\nu=1}^{N_l} f^2(l,\nu)$, where $f^2(l,\nu)=\frac{1}{l}\Sigma_{m=1}^{l} y_{l,\nu}^2(m)$. If $F(l)\sim l^{\alpha}$, the slope of the log $F(l)$ versus log l plot leads to the value of the exponent $\alpha_{DFA}\equiv\alpha$. (This scaling exponent is a self-similarity parameter that represents the long-range power-law correlations of the signal.) If $\alpha_{DFA}=0.5$, there is no correlation and the signal is uncorrelated (white noise); if $\alpha_{DFA}<0.5$, the signal is anticorrelated; and if $\alpha_{DFA}>0.5$, the signal is correlated and specifically the case $\alpha_{DFA}=1.5$ corresponds to the Brownian motion (integrated white noise).

Thus, the procedure proposed here combines three modern methods, i.e., the nonextensive extension of the G-R law together with natural time and DFA. It will be applied in Sec. IV to synthetic seismic data as well as to real seismic data from two different areas, i.e., San Andreas fault system and Japan: in particular the EQs that occurred during the period of 1981–2003 within the area $N_{32}^{37}W_{114}^{122}$ using the Southern California Earthquake catalog will be hereafter called SCEC (see Figs. 2–4 that will be discussed later). Second, the EQs within $N_{25}^{46}E_{125}^{146}$ for the period of 1967–2003 using the Japan Meteorological Agency catalog will be hereafter simply called "Japan" (see Figs. 2–4). The *thresholds* $m\geq 2.0$ and $m\geq 3.5$ have been considered for SCEC and Japan, respectively, for the sake of data completeness.

II. NONEXTENSIVITY AND EARTHQUAKES

The first studies on the analysis of EQs have been made by Abe and co-workers [17,45,46]. In particular, Abe and Suzuki found that the cumulative distribution of interoccurrence times and the distances between successive EQs can be described by the q-exponential distribution with $q>1$ [46] and with $q<1$ [45], respectively, by analyzing Japan and southern California earthquake data. They then proposed a conjecture stating that $q_t+q_s\lesssim 2$, where q_t and q_s represent the values of the q parameter for temporal and spatial distributions, respectively [45]. Interestingly, this conjecture has been found to be verified by the seismic activities in Iran's

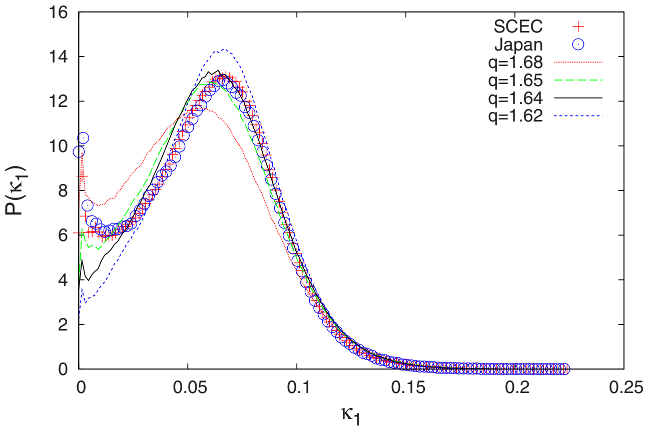

FIG. 2. (Color online) The probability density function $P(\kappa_1)$ versus κ_1 resulting from the natural time analysis of temporally *uncorrelated* data obtained from Eq. (9) for various q values together with those deduced from real seismic data for SCEC and Japan.

faults [47] and reproduced semiquantitatively in recent studies of the two-dimensional Burridge-Knopoff model [48]. Furthermore, Tirnakli and Abe [49] by employing natural time studied the event correlation between aftershocks in the coherent noise model which is an important example and robust mechanism that produces scale-free behavior in the *absence of criticality* (in contrast to the well-known case of self-organized criticality (SOC) [50], where the whole system—under the influence of a small driving force that acts locally—evolves toward a critical stationary state having no characteristic spatiotemporal scales without fine-tuning parameters). Tirnakli and Abe [49] found that the aging phenomenon and the associated scaling property discovered in

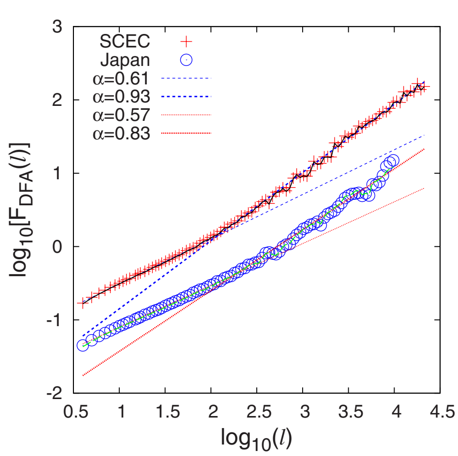

FIG. 3. (Color online) The DFA of the original magnitude time series for SCEC (red plus signs) and Japan (blue circles). The thin and thick straight lines correspond to the short- and the long-scale linear least-squares fit, respectively. The existence of a crossover at $l \approx 200$ leads to the extra complexity in the case of earthquake time series. For this reason, synthetic time series obeying the G-R law [Eq. (2)] with $b=1.08$ have been produced. Their DFA is shown with the thick black (solid) and green (long-dashed) broken lines for SCEC and Japan, respectively. The DFA of Japan has been displaced for the sake of clarity.

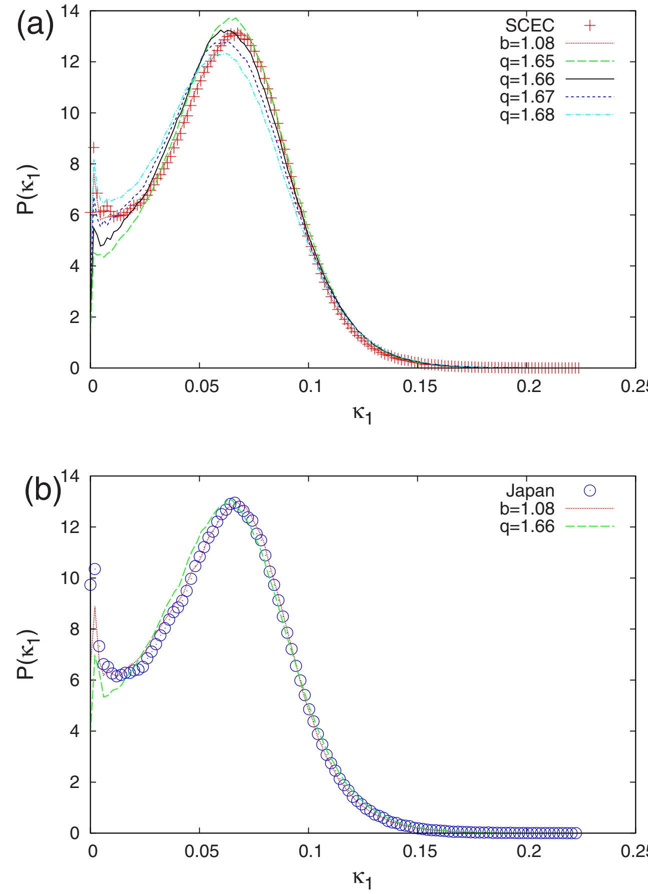

FIG. 4. (Color online) (a) The probability density function $P(\kappa_1)$ versus κ_1 for SCEC (plus signs) together with G-R distributed data with $b=1.08$ (red dotted line) having the same magnitude correlations as the original data [see the black (solid) broken line in Fig. 3]. The green (long-dashed), black (solid), blue (short-dashed), and light blue (dashed-dotted) lines depict $P(\kappa_1)$ versus κ_1 as it results for earthquakes distributed according to Eq. (9) for $q=1.65$, 1.66, 1.67, and 1.68, respectively, when again taking into account the magnitude correlations of the original data. (b) The same as in (a) for Japan (circles), but here only the case of $q=1.66$ (green long-dashed line) is shown.

the observed seismic data are well reproduced by the model. They also found that the scaling function is given by the q-exponential function appearing in nonextensive statistical mechanics showing power-law decay of event correlation in natural time.

An interesting model for earthquake dynamics related to the Tsallis nonextensivity framework has been proposed by Sotolongo-Costa and Posadas (SCP) [51]. It consists basically of two rough profiles interacting via fragments filling the gap between them where the fragments are produced by local breakage of the local plates. In other words, the fundamental idea of this model consists of the fact that the space between faults is filled with the residues of the breakage of the tectonic plates from where the faults originate. In this model the mechanism of earthquake triggering assigns an important role in the fragments: the stress increase between the two fault plates constitutes the main factor that governs the complexity of the fragment-asperity interaction, where eventually the fragments may act as roller bearings, and also as hindering entities of the relative motion of the plates until the growing stress produces their liberation with the subsequent triggering of the earthquake [52]. By using the nonextensive formalism (but see below), SCP not only showed the influence of the size distributions of fragments on the energy distribution of earthquakes but also deduced an energy-distribution function which in a particular case leads to the G-R law.

The aforementioned SCP model was revisited by Silva *et al.* [53] who made two key improvements. The first one made use of a different definition for mean values in the context of Tsallis nonextensive statistics that was achieved in the pioneering study of Ref. [54]. In particular, Abe and Bagci [54] considered in depth the two kinds of definitions for the expectation value of a physical quantity which both lead to the maximum Tsallis entropy distribution of a similar type: the one is the ordinary definition (cf. this was used by SCP) and the other is the normalized q-expectation value employing the escort distribution [55,56]. Their final conclusion states that the Shore-Johnson theorem [57] for consistent minimum cross-entropy (i.e., relative entropy) principle is shown to select the formalism with the normalized q-expectation value and to exclude the possibility of using the ordinary expectation value from nonextensive statistical mechanics. The second improvement by Silva *et al.* refers to the introduction of a scale law, i.e., $\epsilon \sim r^3$, between the released relative energy ϵ and the size r of fragments (This substantially differs from the assumption $\epsilon \sim r$ used by SCP.) Then Silva *et al.* proceeded as follows: the Tsallis entropy has the form

$$S_q = k_B \frac{\int p^q(\sigma)[p(\sigma)^{1-q} - 1]d\sigma}{q-1}, \tag{5}$$

where $p(\sigma)$ is the probability of finding a fragment of relative surface σ (which is defined as a characteristic surface of the system), q is a real number usually termed *nonextensive* parameter, and k_B is the Boltzmann constant which will be hereafter set equal to unity for the sake of simplicity. It is easy to see that Eq. (5) recovers the standard Boltzmann-Gibbs entropy in the limit $q \to 1$. The maximum entropy formulation for Tsallis entropy implies that the following two conditions have to be introduced [18,58]: first, the normalization of $p(\sigma)$:

$$\int_o^\infty p(\sigma)d\sigma = 1, \tag{6}$$

second, the *ad hoc* condition about the q-expectation value:

$$\sigma_q \equiv \langle\sigma\rangle_q = \int_o^\infty \sigma p^q(\sigma)d\sigma, \tag{7}$$

which for $q \to 1$ becomes the definition of the mean value. Silva *et al.* followed the standard method of conditional extremization of the entropy functional S_q and found an expression for the fragment distribution $p(\sigma)$. Then, assuming the aforementioned energy scale $\epsilon \sim r^3$, they obtained the energy-distribution function $p(\epsilon)$ for the EQs. Finally, by considering the relationship

$$m = \tfrac{1}{3}\ln(\epsilon), \tag{8}$$

where m denotes the magnitude, Silva *et al.* obtained the number $N_{>m}$ of EQs with magnitude larger than m:

$$\log\left(\frac{N_{>m}}{N}\right) = \left(\frac{2-q}{1-q}\right)\log\left[1 - \left(\frac{1-q}{2-q}\right)\frac{10^{2m}}{a^{2/3}}\right], \tag{9}$$

where N is the total number of the events and a is the proportionality constant between ϵ and r^3. Equation (9) incorporates the characteristics of nonextensivity into the distribution of earthquakes by magnitude and the G-R law can be deduced as its particular case, i.e., above some magnitude threshold Eq. (9) reduces to Eq. (2) with $b=2(2-q)/(q-1)$. Thus, Eq. (9) can be alternatively termed as a generalized G-R law. This relation has been found to describe appropriately the energy distribution in a wider detectable range of magnitudes compared to that of the original G-R law [59]. Furthermore, Silva *et al.* [53] and later Vilar *et al.* [59] in conjunction with the earlier SCP study [51] led to the conclusion [59] that values of $q \simeq 1.6-1.7$ seem to be universal in the sense that different data sets from different regions of the globe (e.g., California [51], Iberian Peninsula [51], Andalucia [51], Samambaia-Brazil [53], New Madrid, USA [53], North Anatolian fault, Turkey [53], San Andreas fault, USA [59]) indicate a value lying in this interval. In addition, in a very recent study [52], a comparable q value (i.e., $q=1.67$) has been found by analyzing the (tectonic) seismicity in Italy, while a somewhat lower value ($q=1.48$) for the volcanic seismicity in Vesuvius. Finally, we note that very recently [60], an alternative relation has been suggested between the released energy and the surface size of fragments, i.e., $\epsilon \sim \exp(\sigma^{1/\lambda_0})$, where λ_0 is a constant in contrast to the relation $\epsilon \sim \sigma^{1/2}$ proposed by SCP [51] and the relation $\epsilon \sim \sigma^{3/2}$ by Silva *et al.* [53]. This, which has been inspired by the fractal nature of the fragments filling the gaps between adjacent fault plates, leads to a different expression for the distribution of EQs as a function of the magnitude which has a q-exponential form, and the fit with the Iran and California catalogs was found to be good. On the other hand, Eq. (9) has no q-exponential form, but it is preferred to be used in Sec. IV since it has been found to describe well the data in a larger number of seismic regions.

III. NATURAL TIME AND EARTHQUAKES

In a time series consisting of N events, the *natural time* $\chi_k = k/N$ serves as an index [20,21] for the occurrence of the kth event. It is, therefore, smaller than or equal to unity. For the analysis of seismicity, the evolution of the pair (χ_k, E_k) is considered [20,22,61–63], where E_k denotes the seismic energy released during the kth event; see Fig. 1 [cf. this energy—which is itself proportional to the seismic moment M_0—and hence we can use in the vertical axis of Fig. 1(b) either E_k or $(M_0)_k$]. Using $\omega = 2\pi\phi$, and ϕ, the *natural frequency*, the following continuous function $\Phi(\omega)$ was introduced [20–22]:

$$\Phi(\omega) = \frac{\sum_{k=1}^{N} E_k \exp\left(i\omega \frac{k}{N}\right)}{\sum_{n=1}^{N} E_n} = \sum_{k=1}^{N} p_k \exp\left(i\omega \frac{k}{N}\right), \quad (10)$$

where $p_k = E_k / \Sigma_{n=1}^{N} E_n$. A kind of normalized power spectrum $\Pi(\omega)$ can now be defined: $\Pi(\omega) = |\Phi(\omega)|^2$. We focus on the properties of $\Pi(\omega)$ or $\Pi(\phi)$ for natural frequencies ϕ less than 0.5 since in this range of ϕ, $\Pi(\omega)$ or $\Pi(\phi)$ reduces [20–22,61] to a *characteristic function* for the probability distribution p_k in the context of probability theory. According to the probability theory, the moments of a distribution and hence the distribution itself can be approximately determined once the behavior of the characteristic function of the distribution is known around zero. For $\omega \to 0$, a Taylor expansion of $\Pi(\omega)$ leads to [20,21,61] $\Pi(\omega) \approx 1 - \kappa_1 \omega^2$, where $\kappa_1 = \langle \chi^2 \rangle - \langle \chi \rangle^2$ is the variance of natural time. $\Pi(\omega)$ as $\omega \to 0$ (or κ_1) has been shown to be an order parameter for seismicity [63]. For critical dynamics, the following relation holds [20,22,24]:

$$\kappa_1 = 0.070. \quad (11)$$

Furthermore, natural time analysis was found to lead, in general, to the identification of the origin of self-similarity as follows [30]: first, if self-similarity results from the process' memory only, the κ_1 value should change to the κ_1 value of the "uniform" distribution $\kappa_u = 1/12$ for the (randomly) shuffled data. Second, if the self-similarity *exclusively* results from process' increments infinite variance, the $\kappa_{1,p}$ value, at which the probability density function $P(\kappa_1)$ (see below) maximizes, should be the same (but different from κ_u) for the original and the randomly shuffled data. This procedure answers, for example, the fundamental problem of distinguishing between stochastic models characterized by different statistics, e.g., between fractal Gaussian intermittent noise and Lévy-walk intermittent noise, which may equally well reproduce some patterns of a time series [35,64]. When *both* sources of self-similarity may be present in the time series (as in the case of seismicity analyzed here) quantitative conclusions on their relative strength can be obtained on the basis of Eqs. (12) and (13) of Ref. [30] by means of the procedure described in Ref. [41]. In particular, the case of real earthquake data, for which several studies point to the conclusion that exhibit complex patterns of magnitude, spatial, and temporal correlations [65–67] the following procedure was applied (this will be used in the next section): by calculating the κ_1 value in a window of length $l = 6-40$ consecutive events sliding through either the original earthquake catalog or a shuffled one, the probability density function $P(\kappa_1)$ can be constructed. The following results have been obtained for SCEC and Japan: comparing the $\kappa_{1,p}$ values, we find that $\kappa_{1,p} \approx 0.066$ for the original data while $\kappa_{1,p} \approx 0.064$ for the surrogate data. (We clarify that upon shuffling the original data, beyond this decrease in the $\kappa_{1,p}$ value from $\kappa_{1,p} \approx 0.066$ to $\kappa_{1,p} \approx 0.064$, the whole feature of the curve $P(\kappa_1)$ versus κ_1 is changed markedly as can be seen in Fig. 5 of Ref. [30].) Both these $\kappa_{1,p}$ values (with a plausible uncertainty of ± 0.001) differ markedly from the value $\kappa_u = 1/12$. This was interpreted as reflecting that the self-similarity mainly originates from the process' increments infinite variance. Additionally, since the $\kappa_{1,p}$ value of the original EQ data does not greatly differ from the value $\kappa_1 \approx 0.070$ found [30] in infinitely ranged temporal correlations, this indicates the importance of temporal correlations rather than their absence, in the EQ catalogs. In other words, the temporal correlations are responsible for the difference between the value of $\kappa_{1,p} \approx 0.064$ of the surrogate data and the value of $\kappa_{1,p} \approx 0.066$ of the original data. Reference [41] sheds more light on the presence of temporal correlations in seismicity data by considering, beyond the natural time analysis, the correlation function suggested recently in Ref. [67].

IV. RESULTS BY COMBINING NONEXTENSIVITY WITH NATURAL TIME

We first explain the procedure followed here for the generation of synthetic seismic data (magnitude series). The general problem of producing surrogate data sets containing random numbers with a given sample power spectrum and a given distribution of values has been treated in detail in Ref. [68]. Here, we use a simplified method to produce long-ranged correlated (EQ) data (magnitude series) that obey an arbitrary cumulative distribution function $F(x)$. This is based on the well-known random number generator of an arbitrary distribution $F(x)$, described in Ref. [69], as well as on the method suggested in Ref. [66]. Let us first recall that in order to construct [69] a random number generator for the distribution $F(x)$ $(=p)$, we simply need the inverse function $F^{-1}(p)$ $(=x)$; then by inserting a (uncorrelated) random sequence p_i of numbers uniformly distributed in the region (0,1), we can obtain the (uncorrelated) random numbers $x_i = F^{-1}(p_i)$ which are distributed according to the cumulative distribution function $F(x)$. Here, we took advantage of the fact that, at least, for the exponential (G-R law) and the distribution function of Eq. (9), if the sequence p_i is long-range correlated, the same holds for the random numbers x_i $[=F^{-1}(p_i)]$ (see, for example, the broken lines in Fig. 3). For example, if we want to produce a series of random numbers x_i that have a DFA exponent equal to α (<1), we can use $x_i = F^{-1}[\Phi_G(z_i)]$, where $\Phi_G(t)$ is the cumulative distribution function of the standard normal (Gaussian) distribution and z_i is a standard fractional Gaussian noise (i.e., with mean equal to zero and unit standard deviation) with $H = \alpha$ (e.g., see Ref. [70]). Moreover, if we want the generated data to mimic the temporal correlations of some experimental data y_i, then by using their (experimental) cumulative distribution function $\Phi_y(t)$, we can use $x_i = F^{-1}[\Phi_y(y_i)]$. This simple method for the sake of convenience will be hereafter called cumulative distribution function transformation (CDFT).

Figure 2 shows the probability density function $P(\kappa_1)$ versus κ_1 deduced from the natural time analysis of synthetic records of (temporally) uncorrelated $(\alpha = 0.5)$ seismic data obeying the nonextensive generalization of the G-R law, i.e., Eq. (9). Results are given for four different values of q, i.e., $q = 1.62$, 1.64, 1.65, and 1.68, lying in the universal range

$q=1.6-1.7$ suggested by Vilar *et al.* [59]. In the same figure, for the sake of comparison, the results obtained from the real seismic data, i.e., SCEC and Japan, are also plotted. An inspection of this figure shows that the results from synthetic data differ markedly from those of the real data. This reveals that, since in natural time analysis the waiting (interoccurrence) times between EQs do not intervene, temporal correlations do exist in the (magnitude time series of) real seismic data. This is in agreement with the results of Ref. [63] in which we showed that the G-R law cannot fully account for the complexity observed in the real seismic data.

Thus, as a second step, we investigated whether synthetic data obeying Eq. (9) can reproduce the real situation when inserting long-range temporal correlations. To quantify the long-range temporal correlations, in the real seismic data, we depict the DFA plots in Fig. 3 for the original magnitude time series of SCEC (red plus signs) and Japan (blue circles). The thin and the thick straight lines result from a linear least-squares fit to the short $[\log_{10}(l) \leq 2]$ and long $[\log_{10}(l) \geq 2.5]$ scales, respectively, for SCEC (red dotted lines) and Japan (blue short-dashed lines). At the short scales, the values of the slope are $\alpha=0.61(2)$ and 0.57(2) for SCEC and Japan, respectively. These values are comparable to those obtained by Lennartz *et al.* [16] by analyzing the seismic records in regimes of stationary seismic activity in Northern and Southern California (i.e., during periods of both catalogs where large aftershock sequences are missing). At larger scales, a crossover is evident in Fig. 3 at $l \approx 200$ above which the slopes are found to be $\alpha=0.93(3)$ and 0.83(3) for SCEC and Japan, respectively.

The aforementioned DFA behavior (i.e., smaller α value at short scales and larger α at long scales) of the real seismic data was then reproduced by synthetic (obtained from CDFT of the original) seismic data coming from the G-R law with $b \approx 1.08$, the DFA plots of which are shown in Fig. 3 with the broken straight lines (solid black and long-dashed green lines for SCEC and Japan, respectively). Figure 4 depicts $P(\kappa_1)$ versus κ_1 plots for the real seismic data of SCEC [Fig. 4(a)] and Japan [Fig. 4(b)] along with those obtained from these synthetic G-R data (red dotted lines). The good agreement between synthetic and real data probably reflects the fact that only EQs above the magnitude completeness threshold have been considered.

To proceed one step further, synthetic seismic data were deduced by using Eq. (9), instead of the G-R law, and CDFT. In particular, for SCEC by adopting $q=1.65$, 1.66, 1.67, and 1.68 and inserting (by means of the CDFT procedure described in the first paragraph of this section) long-range (temporal) magnitude correlations comparable to those found in real data, i.e., $\alpha=0.57$ and 0.93 for short and long scales, respectively, we obtain the green (long-dashed), black (solid), blue (short-dashed), and light blue (dashed-dotted) lines in Fig. 4(a). We observe that the $q=1.67$ curve is closer to the real data but some differences still remain. As for Japan [see Fig. 4(b)], the synthetic long-range correlated data that come from Eq. (9) with $q=1.66$ (green long-dashed line) with $\alpha=0.61$ and 0.83 for the short and long scales, respectively, exhibit much better agreement with the real ones. This agreement between synthetic and real data can be considered as satisfactory if we recall that there exists a considerable deviation between them in Fig. 2 where the results have been obtained from the natural time analysis of synthetic data that were directly computed from Eq. (9) by ignoring long-range temporal correlations.

V. DISCUSSION

Recapitulating the results obtained in the previous section, as well as those discussed in Refs. [30,41], we can say that natural time analysis of real seismic data for both SCEC and Japan reveals that long-range (temporal) correlations between earthquake magnitudes do exist. This finding, which agrees with the results obtained by independent analyses of real seismic data in Refs. [67,71] through a different procedure, also corroborates with a recent theoretical study by Woodard *et al.* [72] of SOC systems. The latter study shows that the memory of past events (avalanches) is stored in the system profile and that the existence of these correlations contradicts the notion that a SOC time series is simply a *random* superposition of events with sizes distributed as a power law (as has been claimed by several previous studies). This is the notion that was initially interpreted that in SOC systems an event "can occur randomly anywhere at any time and cannot 'know' how large it will become," which was proven in Ref. [72] to be a misconception.

We now discuss the present results in comparison to those deduced in a recent study by Caruso *et al.* [73]. In this interesting study, they performed an analysis of the dissipative Olami-Feder-Christensen (OFC) model [74]—which is a well-known SOC model—on a small-world topology by considering avalanche size differences. Caruso *et al.* found that, when criticality appears, the probability distribution function for the avalanche size differences at different times has fat tails with a q-Gaussian shape. The latter, which is typical of Tsallis statistics [75], has the form $f(x)=A[1+(q-1)x^2/B]^{1/(1-q)}$ which generalizes the standard Gaussian curve depending on the parameters A, B, and on the exponent q (cf. for $q=1$ the normal distribution is recovered, and $q \neq 1$ indicates a departure from Gaussian statistics). In particular, Caruso *et al.* found that the q-Gaussian curve that describes well the OFC behavior in the critical regime yields a value $q=2.0 \pm 0.1$. Note also that the avalanche size λ was taken as equivalent to the energy and that the power-law exponent τ of the avalanche size distribution ($\sim \lambda^{-\tau}$) was determined to be $\tau=1.8 \pm 0.1$. In order to compare their theoretical results with actual seismic data, Caruso *et al.* repeated their analysis for the worldwide catalog (WWS) (comprising 689 000 earthquakes during 2001–2006) and the Northern California Catalog (NCEC) (comprising around 400 000 earthquakes during 1966–2006). By considering the quantity $\exp(m)$ (which is related to the energy of an earthquake, but see also below) as corresponding to the avalanche size λ of the OFC model, they found the exponents $\tau=2.7 \pm 0.2$ and $\tau=3.2 \pm 0.2$ for WWS and NCEC, respectively. In both these actual seismic data sets, Caruso *et al.* found that the probability density function of the energy differences between earthquakes can be fitted by a q-Gaussian curve obtaining an exponent $q=1.75 \pm 0.15$. This q value is comparable (within the errors) with both the q value reported here as well as the q value they found for the OFC model.

Another important finding of Caruso *et al.* [73] is that they reported an interrelation between the following two quantities: first, the power-law exponent τ of the avalanche size distribution and second the q parameter of the q-Gaussian that describes the probability density function $P(\Delta\lambda)$ of the differences $\Delta\lambda$ between the avalanche sizes at time $t+\delta$ and time t (δ is an integer; Caruso *et al.* considered $\delta=1$ for consecutive time steps as in natural time). Under the condition that there is *no correlation* between the sizes of two events, the probability $P(\Delta\lambda)$ of obtaining the difference $\Delta\lambda=\lambda(t+\delta)-\lambda(t)$ is given by [73]

$$P(\Delta\lambda) = K\frac{\epsilon^{-(2\tau-1)}}{2\tau-1}\,{}_2F_1\left(\tau,2\tau-1;2\tau,-\frac{|\Delta|\lambda}{\epsilon}\right), \qquad (12)$$

where K is a normalization factor, ϵ is a small positive value, and ${}_2F_1$ is the hypergeometric function. $P(\Delta\lambda)$, which of course depends on τ, can be approached by means of q-Gaussian with ϵ-independent q value. Caruso *et al.*, by applying Eq. (12) for various τ values, found the following stretched exponential interrelation between q and τ:

$$q = \exp(1.19\tau^{-0.795}). \qquad (13)$$

Caruso *et al.* made an attempt toward investigating whether Eq. (13) is obeyed by the aforementioned values obtained from the analysis of WWS and NCEC. In particular, it was found that the value $q=1.75\pm0.15$ is more or less compatible [in the frame of Eq. (13)] with the τ values, i.e., $\tau=2.7\pm0.2$ and $\tau=3.2\pm0.2$ for WWS and NCEC, respectively, *if* the experimental errors are considered. In view of the latter errors, Caruso *et al.* considered this result with caution since they also noticed that "although temporal [and spatial] correlations among avalanches (earthquakes) do surely exist and a certain degree of statistical predictability is likely possible." This caution is fully justified especially in view of the following: if one uses, instead of the quantity $\exp(m)$ used by Caruso *et al.*, the energy calculated from the seismic moment as described in Ref. [63] or the energy given by Eq. (8), the τ values both for WWS and NCEC would result significantly smaller, thus being in better agreement with the expected range for earthquakes, i.e., when $\tau=\gamma$ and $\gamma\approx1.6-1.7$ as mentioned in Sec. I. By the same token, the q values obtained from $P(\Delta\lambda)$ would be considerably altered. Under these circumstances, it cannot be safely supported that Eq. (13), which is anyhow accurate (see also below), is obeyed by the actual seismicity data, thus precluding the existence of long-range temporal correlations between earthquake magnitudes (energies).

In a later study, Bakar and Tirnakli [76] analyzed the Ehrenfest's dog-flea SOC model. They obtained the value $\tau=1.517$ for the power-law exponent with extreme precision (i.e., within the error bars $\pm1.2\times10^{-5}$). Then the behavior of $P(\Delta\lambda)$ was studied and was numerically shown that it converges to a q-Gaussian with $q=2.35$, a value coming directly (and *a priori*) from Eq. (13). This, as noticed by Bakar and Tirnakli, "constitutes the first reliable verification of Caruso *et al.* relation (13) since due to insufficient data set of earthquakes they were unable to provide clear evidence for their own relation." This was achieved using a simple prototype SOC model (different from the one used by Caruso *et al.*) which can be considered as the first clue on the generality of these results rather than being specific only to this model. In other words, as also noticed by Bakar and Tirnakli, although the important relation (13) was obtained by Caruso *et al.* they could not check its validity since the earthquake data they analyzed were not adequate to obtain the τ value with high precision. Consequently, Caruso *et al.* still used q parameter as a fitting parameter. On the other hand, since the τ value of Bakar and Tirnakli was very accurate, they substituted it into Eq. (13) and found the q value as $q=2.35$. This is the value that one should use in the q-Gaussian to check whether $P(\Delta\lambda)$ resulting from earthquake catalogs can be approached. (We emphasize that in the procedure of Bakar and Tirnakli, who did not investigate earthquake data, q is not a fitting parameter anymore.)

Summarizing our discussion, we can state that the findings of Caruso *et al.* related with the actual seismicity data should not be misinterpreted as unambiguously demonstrating the nonexistence of temporal long-range correlations between earthquake magnitudes. Such correlations, as mentioned in the beginning of this section, do exist as shown here by the results of the natural time analysis of the actual seismic data that are strengthened by recent SOC theoretical investigations [72].

VI. CONCLUSIONS

Here, we investigated the nonextensive generalization of the Gutenberg-Richter (G-R) law. We considered only values of the nonextensive parameter q that have been found in the recent literature to fit well with the real seismic data. The results obtained when combining this generalized law with natural time analysis as well as with DFA show the following (cf. natural time representation does not involve the interoccurrence times between seismic events):

(1) The results of the natural time analysis of the synthetic seismic data obtained from either G-R law or its nonextensive generalization [Eq. (9)] deviate markedly from those of the real seismic data for both SCEC (California) and Japan. This unambiguously reveals that long-range (temporal) correlations between magnitudes exist in the real data sets.

(2) DFA applied to the magnitude time series of the real seismic data demonstrates independently the existence of temporal correlations. The DFA exponent is around 0.6 for short scales but $\alpha=0.8-0.9$ for longer scales (cf. the crossover is noticed around $l\approx200$).

(3) Inspired by point (2), temporal correlations, with different α values (i.e., $\alpha\approx0.6$ and 0.8–0.9 for short and long scales, respectively) were inserted to synthetic seismic data coming from either the G-R law or its nonextensive generalization [Eq. (9)]. The natural time analysis of the correlated synthetic seismic data deduced the from G-R law leads to results that agree well with those obtained from the real seismic data of Japan and SCEC, thus confirming the importance of temporal correlations between the magnitudes of successive earthquakes. As for the synthetic seismic data deduced

from Eq. (9) by inserting long-range temporal correlations, satisfactory agreement with real data has been obtained for the case of Japan for $q=1.66$ while for SCEC some differences still remain.

The present results show that the nonextensive parameter q does not capture the effect of long-range temporal correlations between the magnitudes of successive earthquakes. Thus, published claims (not by the pioneers of the field of nonextensive statistical mechanics) that q is a measure of (temporal) organization do not hold. On the other hand, the generalization of the G-R law, which is a remarkable achievement resulted from Tsallis formulation, when combined with natural time analysis (which focuses on the *sequential order* of the energies of the events that appear in nature) does enable a satisfactory description of the real seismic data fluctuations.

[1] J. B. Rundle, D. L. Turcotte, R. Shcherbakov, W. Klein, and C. Sammis, Rev. Geophys. **41**, 1019 (2003).
[2] H. Kanamori, Nature (London) **271**, 411 (1978).
[3] T. Utsu, Geophys. Mag. **30**, 521 (1961).
[4] R. Shcherbakov, D. L. Turcotte, and J. B. Rundle, Geophys. Res. Lett. **31**, L11613 (2004).
[5] A. Saichev and D. Sornette, Tectonophysics **431**, 7 (2007).
[6] M. Båth, Tectonophysics **2**, 483 (1965).
[7] D. Sornette and P. Davy, Geophys. Res. Lett. **18**, 1079 (1991).
[8] P. Davy, A. Sornette, and D. Sornette, Nature (London) **348**, 56 (1990).
[9] P. Bak, K. Christensen, L. Danon, and T. Scanlon, Phys. Rev. Lett. **88**, 178501 (2002).
[10] D. L. Turcotte, *Fractals and Chaos in Geology and Geophysics*, 2nd ed. (Cambridge University Press, Cambridge, England, 1997).
[11] D. Sornette, *Critical Phenomena in Natural Science*, 2nd ed. (Springer, Berlin, 2004).
[12] J. M. Carlson, J. S. Langer, and B. E. Shaw, Rev. Mod. Phys. **66**, 657 (1994); A. Helmstetter and D. Sornette, J. Geophys. Res. **107**, 2237 (2002); D. S. Fisher, K. Dahmen, S. Ramanathan, and Y. Ben-Zion, Phys. Rev. Lett. **78**, 4885 (1997); J. B. Rundle, W. Klein, and S. Gross, *ibid.* **76**, 4285 (1996); W. Klein, J. B. Rundle, and C. D. Ferguson, *ibid.* **78**, 3793 (1997).
[13] J. R. Holliday, J. B. Rundle, D. L. Turcotte, W. Klein, K. F. Tiampo, and A. Donnellan, Phys. Rev. Lett. **97**, 238501 (2006).
[14] S. Lennartz, A. Bunde, and D. L. Turcotte, Phys. Rev. E **78**, 041115 (2008).
[15] A. S. Balankin, Phys. Rev. E **76**, 056120 (2007); A. S. Balankin, D. Morales Matamoros, J. Patiño Ortiz, M. Patiño Ortiz, E. Pineda León, and D. Samayoa Ocha, EPL **85**, 39001 (2009); J. Davidsen, P. Grassberger, and M. Paczuski, Phys. Rev. E **77**, 066104 (2008).
[16] S. Lennartz, V. N. Livina, A. Bunde, and S. Havlin, EPL **81**, 69001 (2008).
[17] S. Abe and Y. Okamoto (Eds.), *Nonextensive Statistical Mechanics and Its Applications* (Springer, Berlin, 2001).
[18] C. Tsallis, *Introduction to Nonextensive Statistical Mechanics* (Springer, Berlin, 2009).
[19] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
[20] P. A. Varotsos, N. V. Sarlis, and E. S. Skordas, Practica of Athens Academy **76**, 294 (2001).
[21] P. A. Varotsos, N. V. Sarlis, and E. S. Skordas, Phys. Rev. E **66**, 011902 (2002).
[22] P. A. Varotsos, N. V. Sarlis, and E. S. Skordas, Acta Geophys. Pol. **50**, 337 (2002).
[23] P. A. Varotsos, N. V. Sarlis, and E. S. Skordas, Phys. Rev. E **67**, 021109 (2003); **68**, 031106 (2003).
[24] N. V. Sarlis, E. S. Skordas, M. S. Lazaridou, and P. A. Varotsos, Proc. Jpn. Acad., Ser. B: Phys. Biol. Sci. **84**, 331 (2008).
[25] P. A. Varotsos, N. V. Sarlis, E. S. Skordas, and M. S. Lazaridou, Phys. Rev. E **70**, 011106 (2004); **71**, 011110 (2005); Appl. Phys. Lett. **91**, 064106 (2007).
[26] N. V. Sarlis, P. A. Varotsos, and E. S. Skordas, Phys. Rev. B **73**, 054504 (2006).
[27] E. Wigner, Phys. Rev. **40**, 749 (1932).
[28] S. Abe, N. V. Sarlis, E. S. Skordas, H. K. Tanaka, and P. A. Varotsos, Phys. Rev. Lett. **94**, 170601 (2005).
[29] L. Cohen, *Time-Frequency Analysis. Theory and Applications* (Prentice-Hall, Upper Saddle River, NJ, 1994).
[30] P. A. Varotsos, N. V. Sarlis, E. S. Skordas, H. K. Tanaka, and M. S. Lazaridou, Phys. Rev. E **74**, 021123 (2006).
[31] C. K. Peng, S. Havlin, and H. E. Stanley, Chaos **5**, 82 (1995); T. Kalisky, Y. Ashkenazy, and S. Havlin, Phys. Rev. E **72**, 011913 (2005).
[32] J. W. Lamperti, Trans. Am. Math. Soc. **104**, 62 (1962).
[33] A. Weron, K. Burnecki, S. Mercik, and K. Weron, Phys. Rev. E **71**, 016113 (2005).
[34] C. Tsallis, S. V. F. Levy, A. M. C. Souza, and R. Maynard, Phys. Rev. Lett. **75**, 3589 (1995); **77**, 5442 (1996).
[35] N. Scafetta and B. J. West, Complexity **10**, 51 (2005).
[36] P. Varotsos and K. Alexopoulos, Tectonophysics **110**, 73 (1984); **110**, 99 (1984); P. Varotsos and M. Lazaridou, *ibid.* **188**, 321 (1991); P. Varotsos, K. Alexopoulos, and M. Lazaridou, *ibid.* **224**, 1 (1993); P. A. Varotsos, N. V. Sarlis, and E. S. Skordas, Phys. Rev. Lett. **91**, 148501 (2003).
[37] P. Varotsos and K. Alexopoulos, *Thermodynamics of Point Defects and Their Relation with Bulk Properties* (North Holland, Amsterdam, 1986).
[38] P. Varotsos, N. Sarlis, M. Lazaridou, and P. Kapiris, J. Appl. Phys. **83**, 60 (1998).
[39] P. Varotsos, Phys. Rev. B **13**, 938 (1976); J. Appl. Phys. **101**, 123503 (2007); P. Varotsos and K. Alexopoulos, J. Phys. Chem. Solids **39**, 759 (1978); J. Phys. C **12**, L761 (1979); Phys. Rev. B **30**, 7305 (1984).
[40] M. Ausloos and R. Lambiotte, Phys. Rev. E **73**, 011105 (2006).
[41] N. V. Sarlis, E. S. Skordas, and P. A. Varotsos, Phys. Rev. E **80**, 022102 (2009).
[42] C.-K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, and A. L. Goldberger, Phys. Rev. E **49**, 1685 (1994).
[43] M. S. Taqqu, V. Teverovsky, and W. Willinger, Fractals **3**, 785

(1995).
[44] Q. D. Y. Ma, R. P. Bartsch, P. Bernaola-Galván, M. Yoneyama, and P. C. Ivanov, Phys. Rev. E **81**, 031101 (2010); A. Bashan, R. Bartsch, J. W. Kantelhardt, and S. Havlin, Physica A **387**, 5080 (2008).
[45] S. Abe and N. Suzuki, J. Geophys. Res. **108**, 2113 (2003).
[46] S. Abe and N. Suzuki, Physica A **350**, 588 (2005).
[47] A. H. Darooneh and C. Dadashinia, Physica A **387**, 3647 (2008).
[48] T. Hasumi, Phys. Rev. E **76**, 026117 (2007); Physica A **388**, 477 (2009).
[49] U. Tirnakli and S. Abe, Phys. Rev. E **70**, 056120 (2004).
[50] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
[51] O. Sotolongo-Costa and A. Posadas, Phys. Rev. Lett. **92**, 048501 (2004).
[52] L. Telesca, Physica A **389**, 1911 (2010).
[53] R. Silva, G. S. França, C. S. Vilar, and J. S. Alcaniz, Phys. Rev. E **73**, 026102 (2006).
[54] S. Abe and G. B. Bagci, Phys. Rev. E **71**, 016139 (2005).
[55] C. Beck and F. Schlögl, *Thermodynamics of Chaotic Systems: An Introduction* (Cambridge University Press, Cambridge, England, 1993).
[56] S. Abe and N. Suzuki, Phys. Rev. E **67**, 016106 (2003).
[57] J. E. Shore and R. W. Johnson, IEEE Trans. Inf. Theory **IT-26**, 26 (1980); **IT-27**, 472 (1981); **IT-29**, 942 (1983).
[58] C. Tsallis, R. S. Mendes, and A. R. Plastino, Physica A **261**, 534 (1998).
[59] C. S. Vilar, G. S. França, R. Silva, and J. S. Alcaniz, Physica A **377**, 285 (2007).
[60] A. H. Darooneh and A. Mehri, Physica A **389**, 509 (2010).
[61] P. Varotsos, *The Physics of Seismic Electric Signals* (TERRAPUB, Tokyo, 2005).
[62] H. K. Tanaka, P. V. Varotsos, N. V. Sarlis, and E. S. Skordas, Proc. Jpn. Acad., Ser. B: Phys. Biol. Sci. **80**, 283 (2004).
[63] P. A. Varotsos, N. V. Sarlis, H. K. Tanaka, and E. S. Skordas, Phys. Rev. E **72**, 041103 (2005).
[64] N. Scafetta and B. J. West, Phys. Rev. Lett. **92**, 138501 (2004).
[65] A. Corral, Phys. Rev. Lett. **92**, 108501 (2004); V. N. Livina, S. Havlin, and A. Bunde, *ibid.* **95**, 208501 (2005); A. Saichev and D. Sornette, Phys. Rev. E **72**, 056122 (2005); Phys. Rev. Lett. **97**, 078501 (2006); J. Davidsen and M. Paczuski, *ibid.* **94**, 048501 (2005).
[66] J. F. Eichner, J. W. Kantelhardt, A. Bunde, and S. Havlin, Phys. Rev. E **75**, 011128 (2007).
[67] E. Lippiello, C. Godano, and L. de Arcangelis, Phys. Rev. Lett. **98**, 098501 (2007).
[68] T. Schreiber and A. Schmitz, Phys. Rev. Lett. **77**, 635 (1996).
[69] Particle Data Group, R. M. Barnett, C. D. Carone, D. E. Groom, T. G. Trippe, C. G. Wohl, B. Armstrong, P. S. Gee, G. S. Wagman, F. James, M. Mangano, K. Monig, L. Montanet, J. L. Feng, H. Murayama, J. J. Hernandez, A. Manohar, M. Aguilar-Benitez, C. Caso, R. L. Crawford, M. Rocs, N. A. Tornqvist, K. G. Hayes, K. Hagiwara, K. Nakamura, M. Tanabashi, K. Olive, K. Honscheid, P. R. Burchat, R. E. Shrock, S. Eidelman, R. H. Schindler, A. Gurtu, K. Hikasa, G. Conforto, R. L. Workman, C. Grab, and C. Amsler, Phys. Rev. D **54**, 1 (1996).
[70] P. A. Varotsos, N. V. Sarlis, E. S. Skordas, H. K. Tanaka, and M. S. Lazaridou, Phys. Rev. E **73**, 031114 (2006).
[71] E. Lippiello, L. de Arcangelis, and C. Godano, Phys. Rev. Lett. **100**, 038501 (2008).
[72] R. Woodard, D. E. Newman, R. Sánchez, and B. A. Carreras, Physica A **373**, 215 (2007).
[73] F. Caruso, A. Pluchino, V. Latora, S. Vinciguerra, and A. Rapisarda, Phys. Rev. E **75**, 055101 (2007).
[74] Z. Olami, Hans Jacob S. Feder, and K. Christensen, Phys. Rev. Lett. **68**, 1244 (1992).
[75] C. Tsallis, M. Gell-Mann, and Y. Sato, Europhys. News **36**, 186 (2005).
[76] B. Bakar and U. Tirnakli, Phys. Rev. E **79**, 040103 (2009).