Statistical Significance of Minimum of the Order Parameter Fluctuations of Seismicity Before Major Earthquakes in Japan

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Abstract—In a previous publication, the seismicity of Japan from 1 January 1984 to 11 March 2011 (the time of the M9 Tohoku earthquake occurrence) has been analyzed in a time domain called natural time $\chi$. The order parameter of seismicity in this time domain is the variance of $\chi$ weighted for normalized energy of each earthquake. It was found that the fluctuations of the order parameter of seismicity exhibit 15 distinct minima—deeper than a certain threshold—1 to around 3 months before the occurrence of large earthquakes that occurred in Japan during 1984–2011. Six (out of 15) of these minima were followed by all the shallow earthquakes of magnitude 7.6 or larger during the whole period studied. Here, we show that the probability to achieve the latter result by chance is of the order of $10^{-5}$. This conclusion is strengthened by employing also the receiver operating characteristics technique.

Key words: Natural time analysis, Japan, receiver operating characteristics, Monte Carlo calculation, fluctuations, order parameter of seismicity.

1. Introduction

Earthquakes (EQs) exhibit complex correlations in time, space, and magnitude (TELESCA et al. 2002; EICHNER et al. 2007; HUANG 2008, 2011; TELESCA and LOVALLO 2009; TELESCA 2010; LIPPIELLO et al. 2009, 2012; LENNARTZ et al. 2008, 2011; SARLIS 2011; RUNDLE et al. 2012; TENENBAUM et al. 2012; SARLIS and CHRISTOPOULOS 2012). The EQ scaling laws (TURCOTTE 1997) point to the view (e.g., HOLLIDAY et al. 2006) that a mainshock occurrence may be considered an approach to a critical point. Following this view, VAROTSOS et al. (2005) (see also SARLIS et al. 2008; VAROTSOS et al. 2011b) suggested that the variance $\kappa^2$ of the natural time $\chi$ (see Sect. 2) may serve as an order parameter for seismicity.

The study of the fluctuations of this order parameter, denoted $\beta$ (SARLIS et al. 2010), becomes of major importance near the critical point, i.e., near the mainshock occurrence. We assume that a few months represent the period near criticality before each mainshock (VAROTSOS et al. 2011a, 2012a, b, 2013) motivated by the following aspect: the lead time of seismic electric signals (SES) activities that are considered to be emitted when the system enters the critical stage (VAROTSOS and ALEXOPOULOS 1986; VAROTSOS et al. 1993) ranges from a few weeks to a few months (VAROTSOS and LAZARIDOU 1991; TELESCA et al. 2009, 2010; VAROTSOS et al. 2011b).

Recently, the natural time analysis of seismicity in Japan has been investigated (SARLIS et al. 2013) using the Japan Meteorological Agency (JMA) seismic catalog and considering all the 47,204 EQs of magnitude $M_{JMA} \geq 3.5$ in the period from 1984 to the time of the M9 Tohoku EQ (i.e., 11 March 2011), within the area $25^\circ$–$46^\circ$N, $125^\circ$–$148^\circ$E depicted in Fig. 1. It was found that the fluctuations of the order parameter of seismicity exhibit 15 distinct minima—deeper than a certain threshold—1 to around 3 months before the occurrence of large earthquakes. Six (out of 15) of these minima were followed by all the shallow earthquakes of magnitude 7.6 or larger during the whole period studied (their epicenters are shown in Fig. 1). Among the minima, the minimum before the M9 Tohoku EQ was the deepest [this EQ was also preceded by a seismic quiescence, as found by HUANG and DING (2012) through an improved region-time-length algorithm]. It is the scope of the present paper to investigate the statistical significance of these results obtained by SARLIS et al. (2013).
Natural time analysis has been shown (Abe et al. 2005) to extract the maximum information possible from a given time series. For a time series comprising $N$ events, we define the natural time for the occurrence of the $k$th event of energy $Q_k$ by $v_k = k/N$ (Varotsos et al. 2001, 2002). We then study the evolution of the pair $(v_k, p_k)$ where

$$p_k = Q_k / \sum_{n=1}^{N} Q_n$$

is the normalized energy and construct the quantity $\kappa_1$ which is the variance of $\chi$ weighted by $p_k$

$$\kappa_1 = \sum_{k=1}^{N} p_k \chi_k^2 - \left( \sum_{k=1}^{N} p_k \chi_k \right)^2 \equiv \langle \chi^2 \rangle - \langle \chi \rangle^2,$$

where the quantity $Q_k$—see Eq. (1)—is estimated by means of the usual relation (Kanamori 1978)

$$Q_k \propto 10^{1.5M_l},$$

(e.g., Varotsos et al. 2005, 2011a, 2012a, b, c; Sarlis et al. 2010; Ramírez-Rojas and Flores-Márquez 2013; Varotsos et al. 2013; Flores-Márquez et al. 2014).

The detailed procedure for the computation of $\beta$ has been described by Sarlis et al. (2013). In short, they considered excerpts of the JMA catalog
comprising \( W \) consecutive EQs and defined the quantity \( \beta_W \equiv \frac{\sigma(\kappa_1)}{\mu(\kappa_1)} \) as the variability of the order parameter \( \kappa_1 = \mu(\kappa_1) \) and \( \sigma(\kappa_1) \) stand for the average value and the standard deviation of the \( \kappa_1 \) values for this excerpt of length \( W \). For such an excerpt, we form its subexcerpts consisting of the \( n \)th to \((n + 5)\)th EQs, \((n = 1, 2, \ldots, W - 5)\) and compute \( \kappa_1 \) for each of them by assigning \( \lambda_k = k/6 \) and \( \rho_k = Q_k/\sum_{n=1}^{6} Q_n \), \( k = 1, 2, \ldots, 6 \), to the \( k \)th member of the subexcerpt (since at least \( l = 6 \) EQs are needed for obtaining reliable \( \kappa_1 \)). We iterate the same process for new subexcerpts comprising \( l = 7 \) members, 8 members, ...and finally \( W \) members. Then, we compute the average and the standard deviation of the thus-obtained ensemble of \( \kappa_1 \) values [examples of the \( \kappa_1 \) values resulted from subexcerpts comprising \( l = 6, 40, 100, 200, \) and 300 members (EQs) are given in Fig. 2 for the last \( \approx 10 \) year period before the \( M9 \) Tohoku EQ]. The \( \beta_W \) value for this excerpt \( W \) was assigned to the \((W + 1)\)th EQ in the catalog, the target EQ. Hence, for the \( \beta_W \) value of a target EQ, only its past EQs are used in the calculation. The time evolution of the \( \beta \) value was then pursued by sliding the excerpt \( W \) through the EQ catalog. Since \( \approx 10^5 \) EQs with \( M_{JMA} \geq 3.5 \) occur per month on average, the values \( W = 200 \) and \( W = 300 \) were chosen, which would cover a period of a few months before each target EQ. As an example, we depict in Fig. 3 the values of \( \beta_{200} \) and \( \beta_{300} \) (red, left scale) along with all \( M_{JMA} \geq 6 \) EQs (black, right scale) versus the conventional time during the \( \approx 10 \)-year period before the \( M9 \) Tohoku EQ, i.e., since 1 January 2001 until 11 March 2011. The corresponding \( \beta \) values for the remaining period, i.e., since 1 January 1984 until 31 December 2010, can be visualized in SARLIS et al. (2013). Distinct minima of \( \beta_{200} \) and \( \beta_{300} \)—deeper than a certain threshold and having a ratio \( \beta_{300}/\beta_{200} \) close to unity (in the range of 0.95 to 1.08)—have been identified \( 1–3 \) months before all shallow EQs with magnitude 7.6 or larger in the period from 1984 to the time of the \( M9 \) Tohoku EQ. Especially, the minima of \( \beta_{200} \) precede the EQ occurrence by a lead time \( \Delta_{200} \) which is at the most 96 days (cf. the entries in bold in the first and the third column of Table 1, see also Table 1 of SARLIS et al. 2013). Moreover, nine additional similar minima have been identified (see Table 2 of SARLIS et al. 2013) during the same period, which were followed by large EQs of smaller magnitude within 3 months. These 15 \((=6 + 9)\) minima are summarized here in Table 1.

3. Statistical Evaluation by Means of Monte Carlo

The above 15 \( \beta \) minima have been identified during the \( \approx 27 \)-year study period comprising

<table>
<thead>
<tr>
<th>Date of ( \beta_{200} ) minimum</th>
<th>Value of ( \beta_{200} ) minimum</th>
<th>EQ date</th>
<th>Lat. (°N)</th>
<th>Long. (°E)</th>
<th>( M )</th>
<th>( \Delta_{200} ) (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-10-1986</td>
<td>0.254</td>
<td>14-01-1987</td>
<td>42.45</td>
<td>142.93</td>
<td>6.6</td>
<td>3</td>
</tr>
<tr>
<td>08-08-1989</td>
<td>0.278</td>
<td>02-11-1989</td>
<td>39.86</td>
<td>143.05</td>
<td>7.1</td>
<td>3</td>
</tr>
<tr>
<td>05-04-1992</td>
<td>0.250</td>
<td>18-07-1992</td>
<td>39.37</td>
<td>143.67</td>
<td>6.9</td>
<td>3</td>
</tr>
<tr>
<td>23-05-1993</td>
<td>0.293</td>
<td>12-07-1993</td>
<td>42.78</td>
<td>139.18</td>
<td>7.8</td>
<td>2</td>
</tr>
<tr>
<td>1993-07-13</td>
<td>0.188</td>
<td>12-10-1993</td>
<td>32.03</td>
<td>138.24</td>
<td>6.9</td>
<td>3</td>
</tr>
<tr>
<td>30-06-1994</td>
<td>0.295</td>
<td>04-10-1994</td>
<td>43.38</td>
<td>147.67</td>
<td>8.2</td>
<td>3</td>
</tr>
<tr>
<td>15-10-1994</td>
<td>0.196</td>
<td>28-12-1994</td>
<td>40.43</td>
<td>143.75</td>
<td>7.6</td>
<td>2–3</td>
</tr>
<tr>
<td>17-02-1998</td>
<td>0.237</td>
<td>31-05-1998</td>
<td>39.03</td>
<td>143.85</td>
<td>6.4</td>
<td>3</td>
</tr>
<tr>
<td>12-04-2000</td>
<td>0.229</td>
<td>01-07-2000</td>
<td>34.19</td>
<td>139.19</td>
<td>6.5</td>
<td>3</td>
</tr>
<tr>
<td>09-07-2000</td>
<td>0.243</td>
<td>06-10-2000</td>
<td>35.27</td>
<td>133.35</td>
<td>7.3</td>
<td>3</td>
</tr>
<tr>
<td>12-05-2002</td>
<td>0.244</td>
<td>29-06-2002</td>
<td>43.50</td>
<td>131.39</td>
<td>7.0</td>
<td>2</td>
</tr>
<tr>
<td>03-07-2003</td>
<td>0.289</td>
<td>26-09-2003</td>
<td>41.78</td>
<td>144.08</td>
<td>8.0</td>
<td>3</td>
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<tr>
<td>11-06-2005</td>
<td>0.286</td>
<td>16-08-2005</td>
<td>38.15</td>
<td>142.28</td>
<td>7.2</td>
<td>2</td>
</tr>
<tr>
<td>30-11-2010</td>
<td>0.232</td>
<td>22-12-2010</td>
<td>27.05</td>
<td>143.94</td>
<td>7.8</td>
<td>1</td>
</tr>
<tr>
<td>05-01-2011</td>
<td>0.157</td>
<td>11-03-2011</td>
<td>38.10</td>
<td>142.86</td>
<td>9.0</td>
<td>2</td>
</tr>
</tbody>
</table>

The six cases that were followed by all the shallow EQs of magnitude 7.6 or larger are shown in bold.
Figure 2
Plots showing how the $\kappa_1$ values (left scale) fluctuate during the last ≈10-year period before the $M9$ Tohoku EQ, i.e., since 1 January 2001 until 11 March 2011. Here, we depict examples of the $\kappa_1$ values computed from subexcerpts comprising $l = 6$ (a), 40 (b), 100 (c), 200 (d), and 300 (e) EQs. The EQs with $M_{JMA} \geq 6$ (right scale) are also depicted by the vertical black lines ending at circles.
9,931 days. The maximum lead time for \( W = 200 \) was found to be, as mentioned, \( \Delta t_{200} = 96 \) days. Since the \( b_W \) values are calculated after the occurrence of each of the 47,204 EQs, we can estimate the probability \( p_1 \) to obtain by chance a date having a lead time smaller than 97 days before an EQ of magnitude 7.6 or larger by considering the ratio of the EQs that occurred up to 96 days before an EQ of magnitude 7.6 or larger over the total number of the EQs considered. This value results in \( p_1 = \frac{4,768}{47,204} = 0.101 \% \), and hence the probability to obtain at least six such dates when performing 15 (=6+9) attempts can be obtained by the binomial distribution which leads to \( p_{bin} = 0.237 \% \). Of course, this probability does not correspond to the probability to obtain the results of SARLIS et al. (2013) by chance since the six successful dates may not correspond to different EQs of magnitude 7.6 or larger.

In order to quantify the latter probability, we performed a Monte Carlo calculation in which we generated \( 10^6 \) times, 15 uniformly distributed random integers from 1 to 47,204 to select 15 EQs from the JMA catalog, the occurrence dates of which have been compared with the occurrence dates of the six shallow EQs with magnitude 7.6 or larger in order to examine whether all these six EQs have been preceded by randomly selected EQs with a maximum lead time of 96 days. This Monte Carlo calculation has been run \( 10^3 \) times and the corresponding probability is \( p_{MC} = 0.00436(64) \% \) where the number in parenthesis denotes the standard deviation. Thus, we find that the probability to obtain by chance the results found by SARLIS et al. (2013) is of the order of \( 10^{-5} \). We clarify that this probability refers only to the occurrence time of major EQs, while the relevant calculation for an EQ prediction method (e.g., the one based on SES, VAROTSOS and ALEXOPOULOS 1984a, b; VAROTSOS et al. 1988) should also consider the probabilities to obtain the epicentral area and the magnitude of the impending EQs (VAROTSOS et al. 1996a, b).

4. Statistical Evaluation by Means of Receiver Operating Characteristics

A receiver operating characteristics (ROC) graph (Fawcett 2006) is a technique to depict the quality of binary predictions. It is a plot of the hit rate (or true positive rate) versus the false alarm rate (or false
positive rate), as a function of the total rate of alarms, which is tuned by a threshold in the predictor. The hit rate is the ratio of the cases for which the alarm was on and a significant event occurred over the total number of significant events. The false alarm rate is the ratio of the cases for which the alarm was on and no significant event occurred over the total number of non-significant events. Only if the hit rate exceeds the false alarm rate, a predictor is useful [for example, the ROC analysis has been recently used by Teleseca et al., (2014) to discriminate between seismograms of tsunamiigenic and non-tsunamiigenic EQs]. Random predictions generate equal hit and false alarm rate on average (thus, falling on the blue diagonal in Fig. 4 that will be discussed later), and the corresponding ROC curves exhibit fluctuations which depend on the positive \( P \) cases (i.e., the number of significant events) and the negative \( Q \) cases (i.e., the number of non-significant events) to be predicted. The statistical significance of an ROC curve depends (Mason and Graham 2002) on the area under the curve \( A \) in the ROC plane. Mason and Graham (2002) have shown that \( A = 1 - U/(PQ) \), where \( U \) follows the Mann–Whitney U statistics (Mann and Whitney 1947). Very recently, a visualization scheme for the statistical significance of ROC curves has been proposed (Sarlis and Christopoulos 2014). It is based on \( k \)-ellipses which are the envelopes of the confidence ellipses—cf. a point lies outside a confidence ellipse with probability \( \exp(-k/2) \)—obtained when using a random predictor and vary the prediction threshold. These \( k \)-ellipses cover the whole ROC plane and upon using their \( A \) we can have a measure (Sarlis and Christopoulos 2014) of the probability \( p \) to obtain by chance (i.e., using a random predictor) an ROC curve passing through each point of the ROC plane.

In the present case, we divide the whole period covering 27 years and almost 3 months into 109 3-month periods (i.e., \( P + Q = 109 \)) out of which only six included significant events \((P = 6)\). These six significant events were successfully predicted by the aforementioned minima of Table 1 of Sarlis et al. (2013) (written here in bold in Table 1) that preceded all the shallow EQs with magnitude 7.6 or larger during the study period. Hence, the hit rate is 100%. On the other hand, the nine minima which were followed within 3 months by smaller EQs (see Table 2 of Sarlis et al. 2013 which are not marked in bold in Table 1) may be considered false alarms giving rise to a false alarm rate of \( 9/103 \approx 8.74\% \). By using the FORTRAN code VISROC, \( \ell \) provided by Sarlis and Christopoulos (2014) we obtain: (a) the ROC diagram of Fig. 4 in which we depict by the red circle the operation point that corresponds to the results obtained by Sarlis et al. (2013) and (b) the probability \( p \) to obtain this point by chance based on \( k \)-ellipses which results in \( p_{\text{ROC}} = 0.00314\% \). Interestingly, the value of \( p_{\text{ROC}} \) is compatible with \( p_{\text{MC}} \) estimated in the previous section strengthening the conclusion that the probability to obtain the findings of Sarlis et al. (2013) by chance is of the order of \( 10^{-5} \).

5. Conclusions

Recently, the seismicity of Japan was analyzed in natural time from 1 January 1984 to 11 March 2011 using sliding natural time windows of length \( W \) comprising the number of events that would occur in a few months. Fifteen distinct minima of the variability \( \beta \) of the order parameter of seismicity were identified 1–3 months before large EQs. Among these
minima, six were followed by the stronger EQs, namely all the six shallow EQs with $M_{\text{MA}} \geq 7.6$ that occurred in Japan during this $\approx 27$ year period. The probability to obtain the latter result by chance is of the order of $10^{-5}$ as shown here using Monte Carlo calculation. The same conclusion is obtained when using the ROC technique.

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